Coordinating Failures in Product Selection in Monopolistic Competition Models of Innovation

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1. Introduction

Many studies in the economics of innovation are concerned with the question of whether the market generates the right *amount* or *volume* of innovation. In contrast, the question of whether the market generates the right *mix* or *composition* of innovation has been rarely addressed.

For example, consider a monopolistic competition model of Dixit and Stiglitz (1977), which assumes perfect symmetry across all differentiated products that can be potentially developed in an industry.[1] Perfect symmetry ensures that both the social surplus generated by a particular innovation and the monopoly profit earned by its innovator may depend on the amount of innovation, but never on the composition of products developed by others. This assumption thus rules out any possibility that the market mechanism may fail to select the right combination of products. The only remaining question is whether there is too much or too little product variety in market equilibrium.[2]

Of course, perfect symmetry exists only in mathematics and was introduced solely for the analytical convenience. In reality, we cannot ignore the problem of coordinating product selection. For example, different software products are designed for different networks or platforms. Both the social surplus and monopoly profit created by the development of a particular software product may depend on the popularity of different networks among other software products. In the presence of such dependence, the market mechanism may not succeed in coordinating independent software developers into selecting the right set of products.

In this paper, we allow certain asymmetry across products in the standard Dixit-Stiglitz model to address these issues. First, we show that even a small departure from perfect symmetry can gives rise to the possibility of *multiple equilibria*. In different equilibria, different mixes of products are developed. This occurs because even substitutable products become complementary with each other in the presence of more substitutable alternatives. To put it in another way, your usual enemies become your friends in the presence of a bigger enemy. The multiplicity means that there is no decentralized mechanism of ensuring that they coordinate on the right set of products.

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[1] In fact, Dixit and Stiglitz (1977, Section III) considered a particular departure from perfect symmetry, which is different from ours. Their analysis of this case has not attracted much attention in the literature, presumably because it basically replicates the analysis of Spence (1976) in a general equilibrium setting.

[2] To this, Dixit and Stiglitz came up with a striking answer. They showed that, under some knife-edge conditions, the equilibrium product variety is optimal. Of course, this is not a robust feature of the model, and there are many ways in which the optimality can be overturned. Nevertheless, it suggests that one cannot presume whether the market mechanism generates too little or too much innovation in the ideal world with the perfectly symmetric products.

Thus, *coordination failures* can occur in product selection. Second, we show that the welfare cost due to such coordination failures in product selection may dwarf the welfare cost due to monopoly distortions.[3] Arguably, these results suggest that we should be more concerned about facilitating greater coordination about product developments among innovators rather than about the cost and benefit of monopoly power, i.e., whether the monopoly profit provides a sufficient incentive to justify monopoly pricing distortions, which seems the main preoccupation in the economics of innovation.

Within a partial equilibrium framework, Spence (1976) demonstrated that the market mechanism does not necessarily select the right product. He showed that the market tends to favor a product that allows its innovator to appropriate a greater share of the social surplus generated by means of the monopoly profit over a product that generates a larger social surplus.[4] The market failures in his model are not due to coordination failures among independent product developers. Indeed, his model does not have multiple equilibria. What is essential in our model is the interdependent nature of product developments (the social surplus generated by a particular innovation depends on the composition of products developed elsewhere in the economy), not the imperfect appropriability of the social surplus (i.e., how much an innovator could appropriate the social surplus generated by the innovation by means of monopoly profit). To isolate our mechanism from Spence's, we assume that all the products have the identical constant elasticity of demand, which eliminates the market bias pointed out by Spence.[5]

This paper is also related to the literature on network externalities; see Scotchmer (2004, Ch. 10) for a survey. This literature makes extensive use of partial equilibrium models where the social surplus generated by a particular product *is assumed to* depend, in an *ad hoc* way, on the consumption of related products through some network effects. What we are going to show is that, when certain types of asymmetry across products are introduced into the Dixit and Stiglitz model, similar network effects arise endogenously from the very structure of monopolistic competition. In fact, our analysis may be viewed as a micro foundation for network effects.

2. The Dixit-Stiglitz Model without Perfect Symmetry.

We introduce a small departure from perfect symmetry of the differentiated products assumed in the Dixit-Stiglitz (1977) model. We achieve this by considering the situations where products may be divided into several (possibly infinitely many) categories. More specifically, the representative consumer has preferences, given by the following two-tier utility function:

[3]As will be explained in more detail later, multiple equilibria and coordination failures in our model are caused not by the presence of monopoly power but by the incompleteness of the markets. That is to say, unlike in the standard competitive general equilibrium model where the Walrasian auctioneer posts the prices for all the conceivable goods, the consumers in a monopolistic competition economy cannot signal their demand for all the conceivable goods simultaneously.

[4]Indeed, the market failures in product selection would disappear completely in the Spence model if the innovator is allowed to use price discrimination to extract the social surplus fully.

[5]Spence himself showed that, when all the products have the identical constant elasticity of demand, the market selects the right set of products *in his model*. This same assumption does not rule out the market failures in product selection in our model.

(1) $U = V(X_1, X_2, ..., X_J)$ where $X_j = \left\{ \int_{\Omega_j} \left[x(z_j)\right]^{\frac{\sigma-1}{\sigma}} dz_j \right\}^{\frac{\sigma}{\sigma-1}}$ (j = 1, 2, …, J)

where z_j is an index for a differentiated product in category-j, $x(z_j)$ is the consumption of z_j and J is the number of categories, which is a positive integer or can be infinity. The upper-tier function, V, aggregates across all the categories, satisfies the usual properties of preferences, that is, it is strictly increasing, quasi-concave, etc. Within each category, there exists an unlimited range of differentiated goods, but, due to the fixed cost of product development, only a finite range of goods will be actually made available, which is denoted by Ω_j. Within each category, all products enter symmetrically in the CES aggregator, and $\sigma > 1$ is the *direct partial elasticity of substitution* between each pair of products that belong to the same category.

The representative consumer earns its income by supplying inelastically L units of labor, which is the sole factor of production and taken as a numeraire. The representative consumer maximizes its utility subject to the budget constraint,

(2) $\sum_{j=1}^{J} \int_{\Omega_j} p(z_j)x(z_j)dz_j \leq L$,

where $p(z_j)$ is the unit price charged by the producer of z_j. In maximizing its utility, the representative consumer takes not only all the prices offered as given, but it also takes the set of the products available in the market, Ω_j, as given. From this maximization, consumer demands satisfy

(3) $\frac{x(z_j)}{X_j} = \left[\frac{p(z_j)}{P_j}\right]^{-\sigma}$ and $\frac{\partial V}{\partial X_j}\left(X_1, X_2, ..., X_J\right) = \lambda P_j$ (j = 1, 2, …, J)

where λ is the (Lagrange) multiplier and

(4) $P_j = \left\{ \int_{\Omega_j} \left[p(z_j)\right]^{1-\sigma} dz_j \right\}^{\frac{1}{1-\sigma}}$ (j = 1, 2, …, J)

is the aggregate price index for category-j products (i.e., the effective price of the aggregate quantity index, X_j) and the budget constraint (2) can be written to

(5) $\sum_{j=1}^{J} P_j X_j \leq L$.

Anyone can develop any product in category-j by employing F_j units of labor. In addition, it is assumed that manufacturing each category-j product requires m_j units of labor per unit. Because all products within each category share the same constant marginal labor requirement, we can choose, without loss of generality, the unit of products for each category such that $m_j = 1-1/\sigma$ for all j. Thus, the total cost of supplying x units of any product is $(1-1/\sigma)x + F_j$. This normalization greatly reduces the notational burden below.

Even though there is no barrier to entry, the range of potential products is unbounded (within each category), so that no two developers ever develop the same product and compete with each other in equilibrium (even if everyone wants to develop in the same category). In other words, there is no duplication; each product is developed and produced by an independent developer. Thus, each developer acts as a monopolist. However, being infinitesimally small, each developer takes $\{X_j\}_{j=1}^{J}$, $\{P_j\}_{j=1}^{J}$ and λ in eq.

(3), as given. This means that each developer faces the demand curve with its constant price elasticity equal to σ. Hence, each developer will set the price of its own product at $p(z_j) = p_j = [\sigma/(\sigma-1)]m_j = 1$. This implies that, from (4), the aggregate price index for category-j products is equal to

$$(6) \qquad P_j = N_j^{\frac{1}{1-\sigma}}, \qquad (j = 1, 2, \ldots, J)$$

where N_j is the number of category-j products developed in this economy (i.e., the measure of Ω_j). This also implies that every product within the same category is produced by the same amount, $x(z_j) = x_j$, and hence, the aggregate quantity index for category j is given by

$$(7) \qquad X_j = N_j^{\frac{\sigma}{\sigma-1}} x_j \qquad (j = 1, 2, \ldots, J),$$

and eqs. (3) and (5) can be written as

$$(8) \qquad \sum_{j=1}^{J} N_j x_j = L$$

$$(9) \qquad \frac{\partial V}{\partial X_j}\left(N_1^{\frac{\sigma}{\sigma-1}} x_1, N_2^{\frac{\sigma}{\sigma-1}} x_2, \ldots, N_J^{\frac{\sigma}{\sigma-1}} x_J \right) = \lambda \left(N_j\right)^{\frac{1}{1-\sigma}} \qquad (j = 1, 2, \ldots, J).$$

Eqs. (8)-(9) can be solved for $\{x_j\}_{j=1}^{J}$ for each $\{N_j\}_{j=1}^{J}$.

The gross profit earned by a category-j product is equal to $p_j x_j - m_j x_j = x_j/\sigma$, so that the free entry-exit condition is given by the complementary slackness condition.

$$(10) \qquad \pi_j = \frac{x_j}{\sigma} - F_j \le 0;\; N_j \ge 0;\; \pi_j N_j = 0 \qquad (j = 1, 2, \ldots, J).$$

The equilibrium values of $\{x_j\}_{j=1}^{J}$ and $\{N_j\}_{j=1}^{J}$ can be obtained by solving eqs. (8)-(10).

To obtain the closed-form solutions, suppose now that the upper-tier function is a CES:

$$U = V(X_1, X_2, \ldots, X_J) = \left[\sum_{j=1}^{J} \left(B_j X_j\right)^{1-\frac{1}{\theta}} \right]^{\frac{\theta}{\theta-1}},$$

where θ > 1 is the direct partial elasticity of substitution between any two categories.[6] For a finite θ, the consumer demand curve for each product is given by

$$(11) \qquad x(z_j) = \left[\frac{L\left(B_j\right)^{\theta-1}\left(P_j\right)^{\sigma-\theta}}{\sum_{k=1}^{J}\left(B_k\right)^{\theta-1}\left(P_k\right)^{1-\theta}} \right] \left(p(z_j)\right)^{-\sigma} \qquad (j = 1, 2, \ldots, J).$$

From (6) and $p(z_j) = 1$, the equilibrium market size, $\{x_j\}_{j=1}^{J}$, for each $\{N_j\}_{j=1}^{J}$ are given by:

$$(12) \qquad x_j = \frac{L\left(B_j\right)^{\theta-1}\left(N_j\right)^{\frac{\sigma-\theta}{1-\sigma}}}{\sum_{k=1}^{J}\left(B_k\right)^{\theta-1}\left(N_k\right)^{\frac{1-\theta}{1-\sigma}}} \qquad (j = 1, 2, \ldots, J).$$

From (10), the non-positive profit condition for category-j can be written as

[6] Although much of the following analysis would carry over equally for θ ≤ 1, we impose the restriction for θ > 1. This is because we introduce categories to capture asymmetry across products within the same industry. With θ ≤ 1, they would look more like products that belong to different industries.

(13) $$\sum_{k=1}^{J}\beta_k F_k (N_k)^{\frac{1-\theta}{1-\sigma}} \geq \beta_j (N_j)^{\frac{\sigma-\theta}{1-\sigma}} \frac{L}{\sigma}, \qquad (j = 1, 2, \ldots, J).$$

where $\beta_j \equiv (B_j)^{\theta-1}/F_j$ (j = 1, 2, …, J), and its zero profit condition is given by (13) with the equality.

The first example looks at the knife-edge case, in which this model is equivalent to the Dixit and Stiglitz model with the optimal product variety.

Example 1 (θ = σ). In this case, the representative consumer's preferences become $U = \left\{ \sum_{j=1}^{J} \int_{\Omega_j} \left[B_j x(z_j) \right]^{\frac{\sigma-1}{\sigma}} dz_j \right\}^{\frac{\sigma}{\sigma-1}}$. Note that the direct partial elasticity of substitution between any pair of products is equal to σ, whether they belong to the same category or not. From (13), the zero-profit condition for category-j products is simplified to

$$\sum_{k=1}^{J}\beta_k F_k N_k = \beta_j \frac{L}{\sigma}. \qquad (j = 1, 2, \ldots, J).$$

This implies that the ranking of different categories in profitability depends solely on β_j. In particular, it is independent of $\{N_j\}_{j=1}^{J}$. Suppose that J is finite, so that $\beta_* = \text{Max} \{\beta_j\}_{j=1}^{J}$ exist. Then, in equilibrium, only the products in the categories with $\beta_j = \beta_*$ are developed. Each of these products is produced by $x_j = \sigma F_j$, but the distribution of products across the most profitable categories is indeterminate. Furthermore, the representative consumer would enjoy the optimal level of the utility in any equilibrium, $U = (\beta_*/\sigma)^{\frac{1}{\sigma-1}} L^{\frac{\sigma-1}{\sigma}}$, where $\beta_* = \text{Max} \{\beta_j\}_{j=1}^{J}$.

Figures 1A and 1B illustrate this for J = 2. In Figure 1A, $\beta_1 = \beta_2$, so that both categories of products are equally profitable. The thick negative-sloped line, $F_1 N_1 + F_2 N_2 = L/\sigma$, is the zero-profit condition for both categories. Below this line, all products make profits so that more of both categories will be developed, as indicated by upward arrows. Above the line, they all make losses, so that fewer of both categories will be developed, as indicated by downward arrows. At any point on the negative-sloped line, both categories make zero-profit, so that there is a continuum of equilibria. The indeterminacy of equilibrium is, however, without any real consequence. Furthermore, these equilibria are optimal. This should not be surprising, because all products are essentially identical in this case; the categories are merely "labels" without any substance. So, this is effectively a model with only one category of products, bringing it back to the original Dixit and Stiglitz model with the optimal product variety.

Starting from this situation, imagine that a small exogenous change makes category-2 relatively more attractive, i.e., $\beta_1 < \beta_2$. The effect of this change is illustrated by Figure 1B. Now, there are two zero profit conditions, indicated by the two negative-sloped lines. The zero-profit condition for category-1 is given by the dotted line, $\beta_1 F_1 N_1 + \beta_2 F_2 N_2 = \beta_1 L/\sigma$, while the zero-profit condition for category-2 is given by the sold line, $\beta_1 F_1 N_1 + \beta_2 F_2 N_2 = \beta_2 L/\sigma$. Below the dotted line, all products make profits. Above the solid line, all products make losses. Between the two lines, category-1 products make losses and category-2 products make profits. The equilibrium is now given uniquely at $(N_1, N_2) = (0, L/\sigma F_2)$, as indicated by the black circle. In this case, the less profitable category-1 products are abandoned and the economy develops only category-2 products, which again brings this model effectively back to a one-category case, i.e., the original Dixit and Stiglitz model with the optimal product variety.

As this example shows, departing from perfect symmetry by merely introducing heterogeneity in productivity across products would not fundamentally change the property of the model. The less productive products will simply not be developed in equilibrium, and hence will become irrelevant.

The critical feature of the above example (and of the Dixit-Stiglitz model) is that the development of any product would not affect the *relative* attractiveness of developing any other products, because the direct partial elasticity of substitution between any pair of products is the same, regardless of their categories. However, this depends on the knife-edge condition, $\theta = \sigma$, and hence it is not a robust situation, as the next example demonstrates.

Example 2 ($\theta > \sigma$): This is the case where categories are more substitutable than products within each category are. This means that the representative consumer cares more about variety within each category than variety across categories. To put it another way, different categories compete with each other more than different products compete with each other within each category. This case thus captures, among other things, the situation where categories represent competing technologies, industrial standards, platforms or systems, and products in each category share the same standard, or are based on the same system. When different systems compete with each other, different products that share the same system may become complements with each other (in the sense of Hicks-Allen). This can be verified by calculating *the Allen partial elasticity of substitution* from eq. (11) as follows:

$$(14)\qquad \frac{\partial \log x(z_j)}{\partial \log(P_j)} = \sigma - \theta - \frac{(1-\theta)\left(P_j / B_j\right)^{1-\theta}}{\sum_{k=1}^{J}\left(P_k / B_k\right)^{1-\theta}}.$$

In the absence of competing systems (i.e., $P_k = \infty$ for all $k \neq j$), eq. (14) becomes

$$\frac{\partial \log x(z_j)}{\partial \log(P_j)} = \sigma - 1 > 0,$$

so that the demand for a category-j product goes up when the prices of other category-j products go up. Thus, products that share the same system are Hicks-Allen substitutes with each other. In the presence of competing systems, however, this expression could become negative. For example,

$$\frac{\partial \log x(z_j)}{\partial \log(P_j)} \to \sigma - \theta < 0$$

as $P_j \to \infty$. This means that the demand for a category-j products declines when the prices of other category-j products go up. Thus, products that share the same system are

Hicks-Allen complements. Note that this is the case, in spite that the *direct partial elasticity of substitution* is assumed to be high ($\sigma > 1$).[7]

Because products that share the same system are complements in the presence of competing systems, product development in one system makes the other products in the same system more attractive for the representative consumer. This can be verified in eq. (12), which shows that the market size for category-j products is increasing in N_j and decreasing in N_k for $k \neq j$. Thus, product development in one system makes product development in the same system even more profitable. On the other hand, product development in any system is never profitable in the absence of other products that share the same system.

This positive feedback leads to multiple equilibria. From eq. (13), one can easily verify that there are J stable equilibria. At the j-th equilibrium, $x_j = \sigma F_j$, $N_j = L/\sigma F_j$, and $x_k = N_k = 0$ for all $k \neq j$. Thus, the only products that belong to category-j are developed. In other words, the j-th system becomes dominant. The level of utility at this equilibrium is given by

$$U_j = B_j (L^\sigma / \sigma F_j)^{\frac{1}{\sigma - 1}}.$$

Note that any of the J systems can become the dominant system even if it has arbitrarily smaller B or arbitrarily larger F than its competing systems. Furthermore, this multiplicity occurs even if θ is slightly higher than σ.

There are also unstable equilibria. For example, $N_j = 0$ for all j is an equilibrium, because the zero-profit condition for each j is satisfied. However, this equilibrium is unstable, because an arbitrarily small but positive entry into category-j, $N_j = \varepsilon > 0$, would make development of category-j products profitable. Any equilibrium in which two or more categories are developed is also unstable. To see this, suppose that both category-1 and category-2 are developed in equilibrium. Then, the zero-profit conditions must hold for both, so that $\beta_1 (N_1)^{\frac{\sigma-\theta}{1-\sigma}} (L/\sigma) = \sum_{k=1}^{J} \beta_k F_k (N_k)^{\frac{1-\theta}{1-\sigma}} = \beta_2 (N_2)^{\frac{\sigma-\theta}{1-\sigma}} (L/\sigma)$. Now, let us increase N_1 and decrease N_2 slightly simultaneously to keep the mid-term constant. This leads to $\beta_1 (N_1)^{\frac{\sigma-\theta}{1-\sigma}} (L/\sigma) > \sum_{k=1}^{J} \beta_k F_k (N_k)^{\frac{1-\theta}{1-\sigma}} > \beta_2 (N_2)^{\frac{\sigma-\theta}{1-\sigma}} (L/\sigma)$, which implies that category-1 make positive profits and category-2 make negative profits.

Figure 2 illustrates all of this for J = 2. The zero-profit condition for category-1 is represented by the hump-shaped curve,

$$\beta_1 F_1 (N_1)^{\frac{1-\theta}{1-\sigma}} + \beta_2 F_2 (N_2)^{\frac{1-\theta}{1-\sigma}} = \beta_1 (N_1)^{\frac{\sigma-\theta}{1-\sigma}} \frac{L}{\sigma},$$

which starts from the origin and returns to the N_1-axis at $N_1 = L/\sigma F_1$. Below this curve, more category-1 products would be developed, as they would make profits, and above this curve, fewer of them would be developed as they make losses. The zero-profit condition for category-2 is represented by the inverted C-curve,

$$\beta_1 F_1 (N_1)^{\frac{1-\theta}{1-\sigma}} + \beta_2 F_2 (N_2)^{\frac{1-\theta}{1-\sigma}} = \beta_2 (N_2)^{\frac{\sigma-\theta}{1-\sigma}} \frac{L}{\sigma},$$

[7] Recall that the direct partial elasticity of substitution between good A and good B measures how much the demand for A responds to a change in the price of good B, if the demand for other goods are not allowed to change. This is not a particularly useful concept, outside of the two-goods world.

which starts from the origin and returns to the N_2-axis at $N_2 = L/\sigma F_2$. To the left of this curve, more category-2 products would be developed, and to the right of this curve, fewer category-2 products would be developed. These are all indicated by the directions of the arrows.

Two black circles depict the two stable equilibria,

$(N_1, N_2) = (L/\sigma F_1, 0)$ and $(N_1, N_2) = (0, L/\sigma F_2)$.

In the former, category-2 products are not profitable and only category-1 products are developed. In the latter, category-1 products are not profitable and only category-2 products are developed. There are also two unstable equilibria:

$(N_1, N_2) = (0, 0)$

and

$$(N_1, N_2) = \left(\frac{L/\sigma}{F_1 + (\beta_2/\beta_1)^{\frac{1-\sigma}{\theta-\sigma}} F_2}, \quad \frac{L/\sigma}{(\beta_1/\beta_2)^{\frac{1-\sigma}{\theta-\sigma}} F_1 + F_2} \right).$$

As indicated by the arrows, neither of them is stable. Furthermore, the latter has counter-intuitive comparative static properties. For example, a higher B_1/B_2 reduces N_1 and increases N_2.

Let us now focus on the two stable equilibria, depicted by the two black circles. One of them is generally dominated by the other.**

Example 3 ($\theta = \infty$): Eqs. (11)-(13) assume that θ is finite. However, the main results carry over for $\theta = \infty$, $U = \sum_{j=1}^{J} B_j X_j$, i.e., when the competing systems are perfect substitutes. In this case, the representative consumer demand only system-j products, iff $B_j/P_j > B_k/P_k$ for all k, or $B_j N_j^{1/(\sigma-1)} \geq B_k N_k^{1/(\sigma-1)}$ for all k. The zero-profit condition for j becomes $N_j = L/\sigma F_j > (B_k / B_j)^{\sigma-1} N_k$ for all k. Again, there are J stable equilbria. In the J-th equilibrium, $x_j = \sigma F_j$, $N_j = L/\sigma F_j$, $x_k = N_k = 0$ for all k, and $U_j = B_j (L^\sigma / \sigma F_j)^{1/(\sigma-1)}$.

Example 4 ($1 < \theta < \sigma$): This is the case where the representative consumer cares more about variety across categories than variety within each category. This makes products that belong to the same category Hicks-Allen substitutes with each other. This can be verified from Eq. (14), which shows that the Allen partial elasticity of substitution is always positive for $\theta < \sigma$.

Because products that belong to the same category are substitutes, fewer products in one category make product development in that category profitable. Indeed, the RHS of (13) is decreasing in N_j and it goes to infinity as $N_j = 0$, which implies that the equilibrium is in the interior. That is, the zero-profit condition holds for all the categories.

$$\sum_{k=1}^{J} \beta_k F_k (N_k)^{\frac{1-\theta}{1-\sigma}} = \beta_j (N_j)^{\frac{\sigma-\theta}{1-\sigma}} \frac{L}{\sigma} \qquad (j = 1, 2, \ldots, J).$$

In other words, all the categories are developed in the unique equilibrium.

Figure 4 illustrates this for J = 2. The zero-profit condition for category-1,

$$\beta_1 F_1 (N_1)^{\frac{1-\theta}{1-\sigma}} + \beta_2 F_2 (N_2)^{\frac{1-\theta}{1-\sigma}} = \beta_1 (N_1)^{\frac{\sigma-\theta}{1-\sigma}} \frac{L}{\sigma},$$

is now negatively sloped, asymptotic to the N_2-axis, and hits the N_1-axis at $N_1 = L/\sigma F_1$. More category-1 products are developed to the left of this curve, and fewer of them would be developed to its right. Likewise, the zero-profit condition for category-2,

$$\beta_1 F_1 (N_1)^{\frac{1-\theta}{1-\sigma}} + \beta_2 F_2 (N_2)^{\frac{1-\theta}{1-\sigma}} = \beta_2 (N_2)^{\frac{\sigma-\theta}{1-\sigma}} \frac{L}{\sigma},$$

is negatively sloped, asymptotic to the N_1-axis and hits the N_2-axis at $N_2 = L/\sigma F_2$. More category-2 products would be developed below this curve, and fewer of them would be developed above it. These are all indicated by the directions of the arrows. There is a unique equilibrium in the interior, given by

$$(N_1, N_2) = \left(\frac{L/\sigma}{F_1 + (\beta_2/\beta_1)^{\frac{1-\sigma}{\theta-\sigma}} F_2}, \quad \frac{L/\sigma}{(\beta_1/\beta_2)^{\frac{1-\sigma}{\theta-\sigma}} F_1 + F_2} \right).$$

As shown, this unique equilibrium is stable. Furthermore, it has the intuitive comparative static properties.

3. Anatomy of Coordination Failures

(I) $$\sum_{j=1}^{J} \int_{\Omega_j} p(z_j)x(z_j)dz_j \le L + qH$$

(II) $$P_j = N_j^{\frac{1}{1-\sigma}}, \qquad (j = 1, 2, \ldots, J)$$

(III) $$X_j = N_j^{\frac{\sigma}{\sigma-1}} x_j \qquad (j = 1, 2, \ldots, J),$$

(IV) $$\sum_{j=1}^{J} N_j m_j x_j = L$$

(V) $$\sum_{j=1}^{J} N_j F_j = H$$

(VI) $$\frac{\partial V}{\partial X_j}\left(N_1^{\frac{\sigma}{\sigma-1}} x_1, N_2^{\frac{\sigma}{\sigma-1}} x_2, \ldots, N_J^{\frac{\sigma}{\sigma-1}} x_J\right) = \lambda\left(N_j\right)^{\frac{1}{1-\sigma}} \qquad (j = 1, 2, \ldots, J).$$

(VII) $$\pi_j = \frac{x_j}{\sigma} - qF_j \le 0;\ N_j \ge 0;\ \pi_j N_j = 0 \qquad (j = 1, 2, \ldots, J).$$

(IV)-(VI) can be solved for $\{x_j\}_{j=1}^{J}$ for each $\{N_j\}_{j=1}^{J}$ satisfying (V).

References:

Dixit and Stiglitz (1977)
Scotchmer (2004)
Spence (1976)

Figure 1A: $\theta > \sigma$.

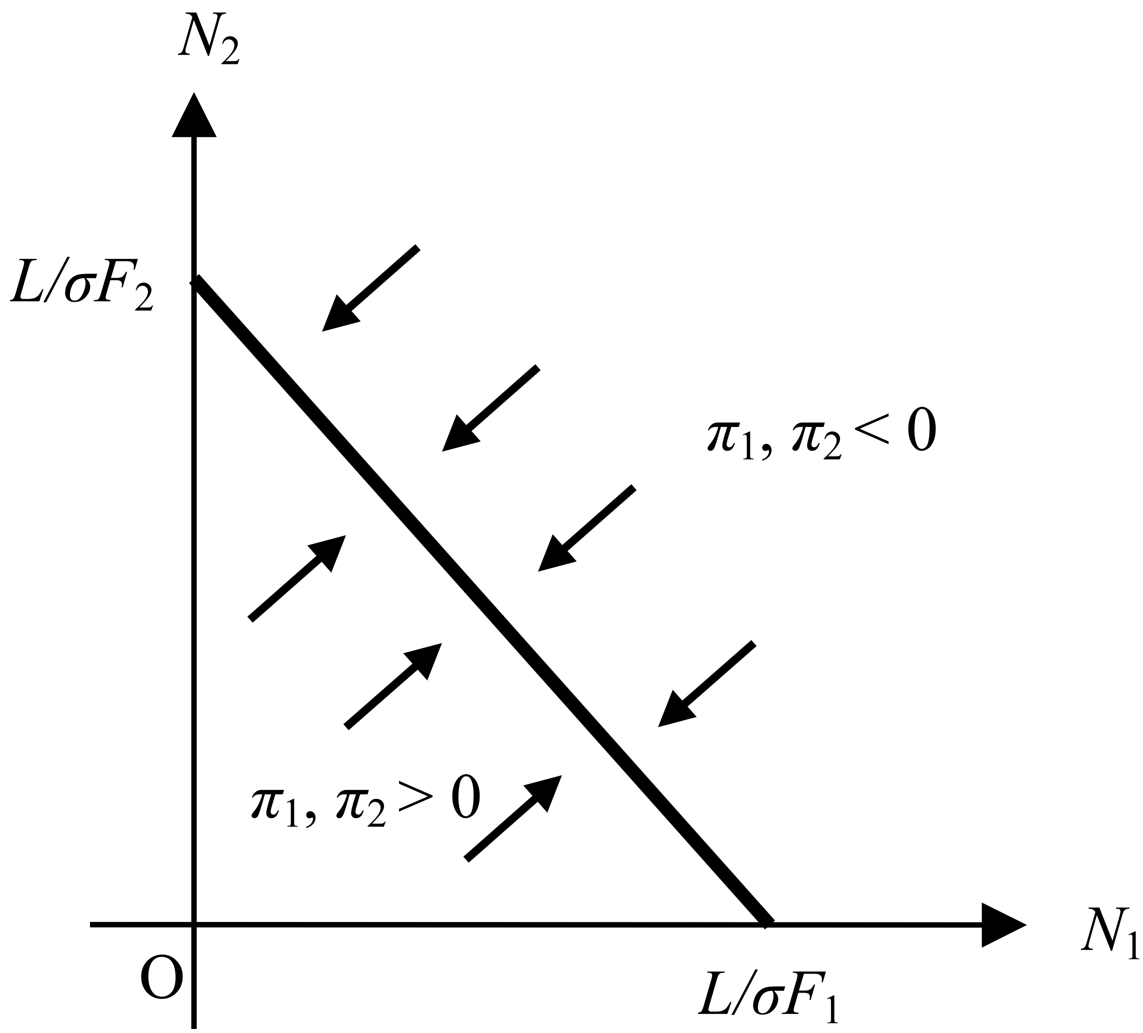

Figure 1B: θ > σ.

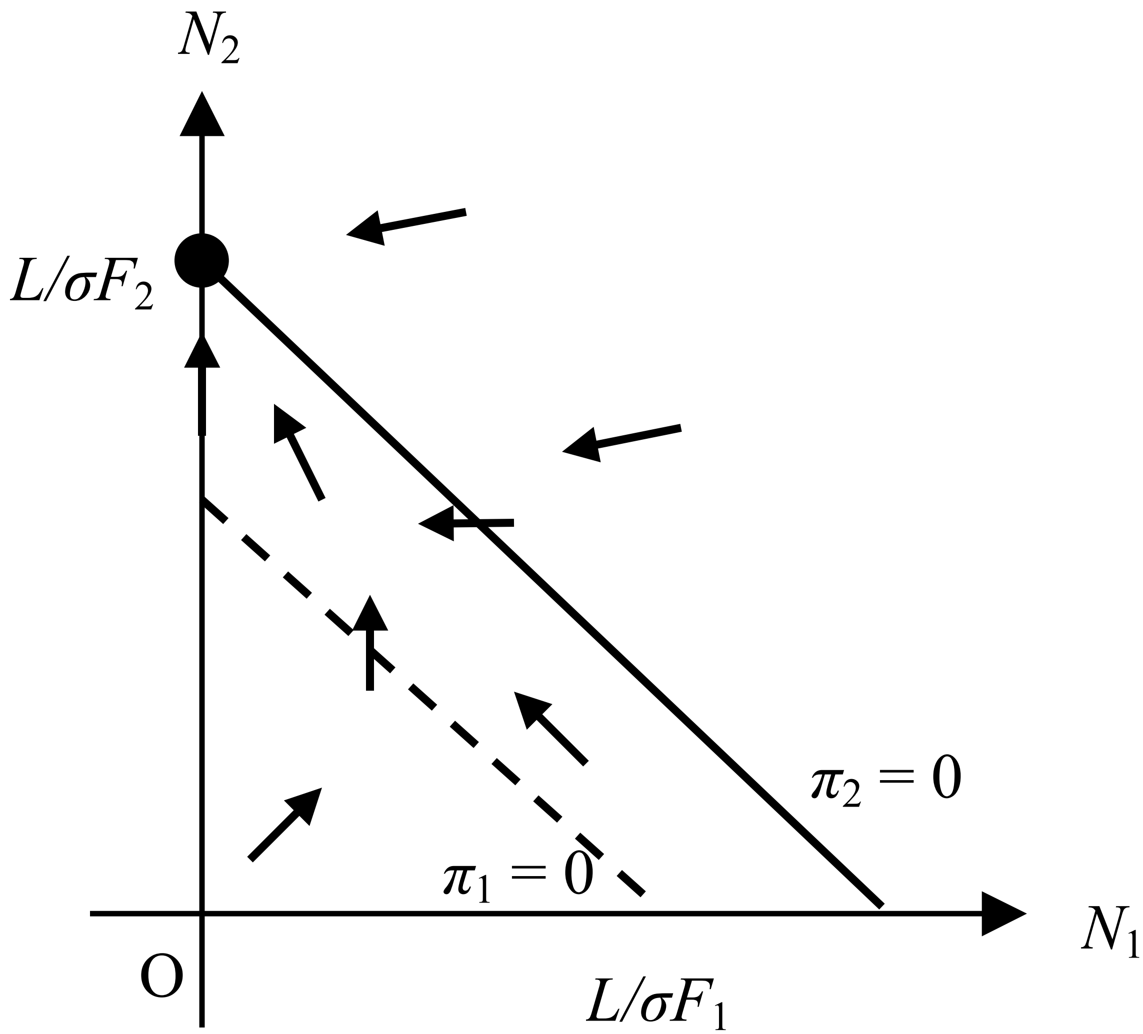

Figure 2: θ > σ.

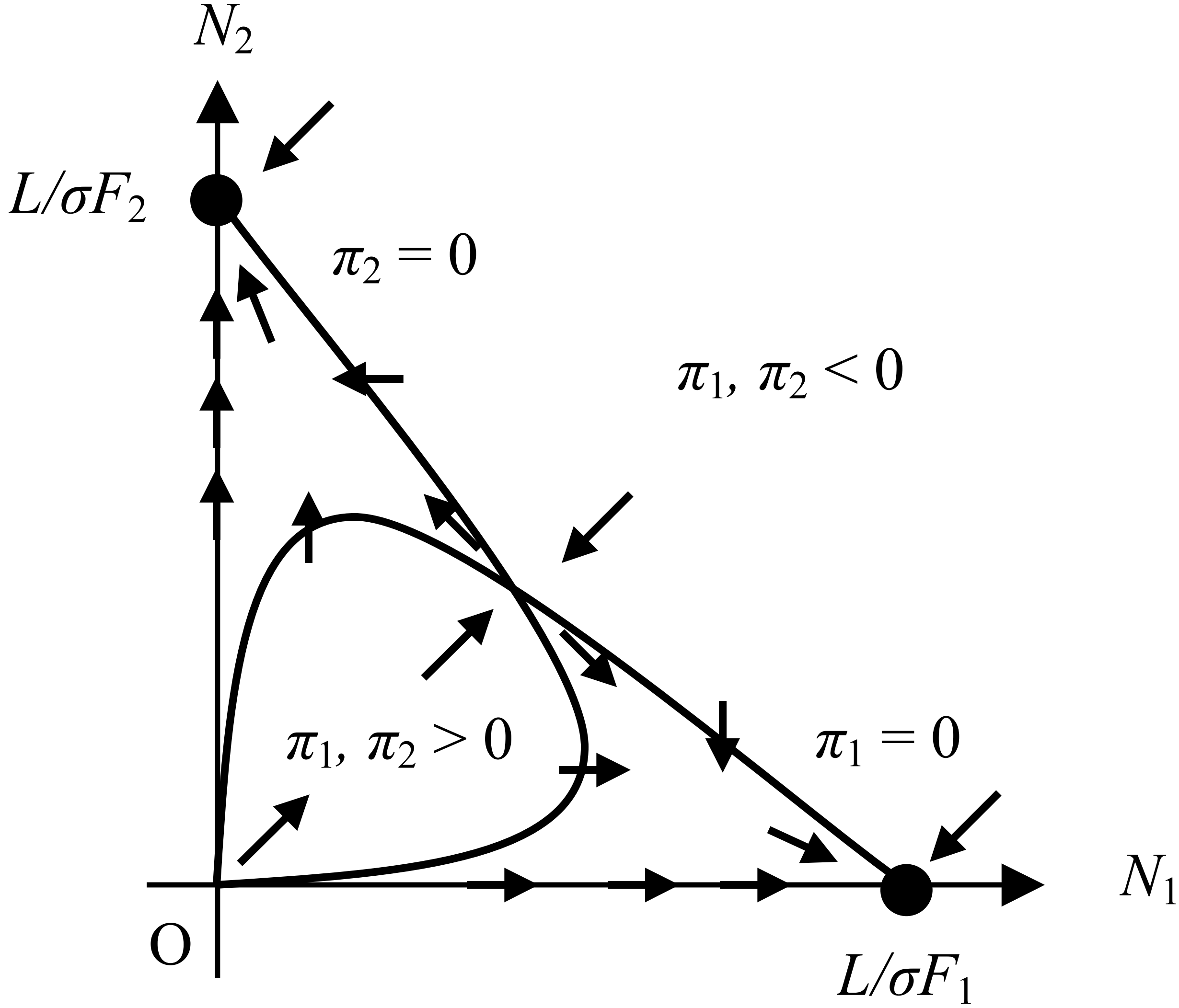

Figure 3: $1 < \theta < \sigma$.

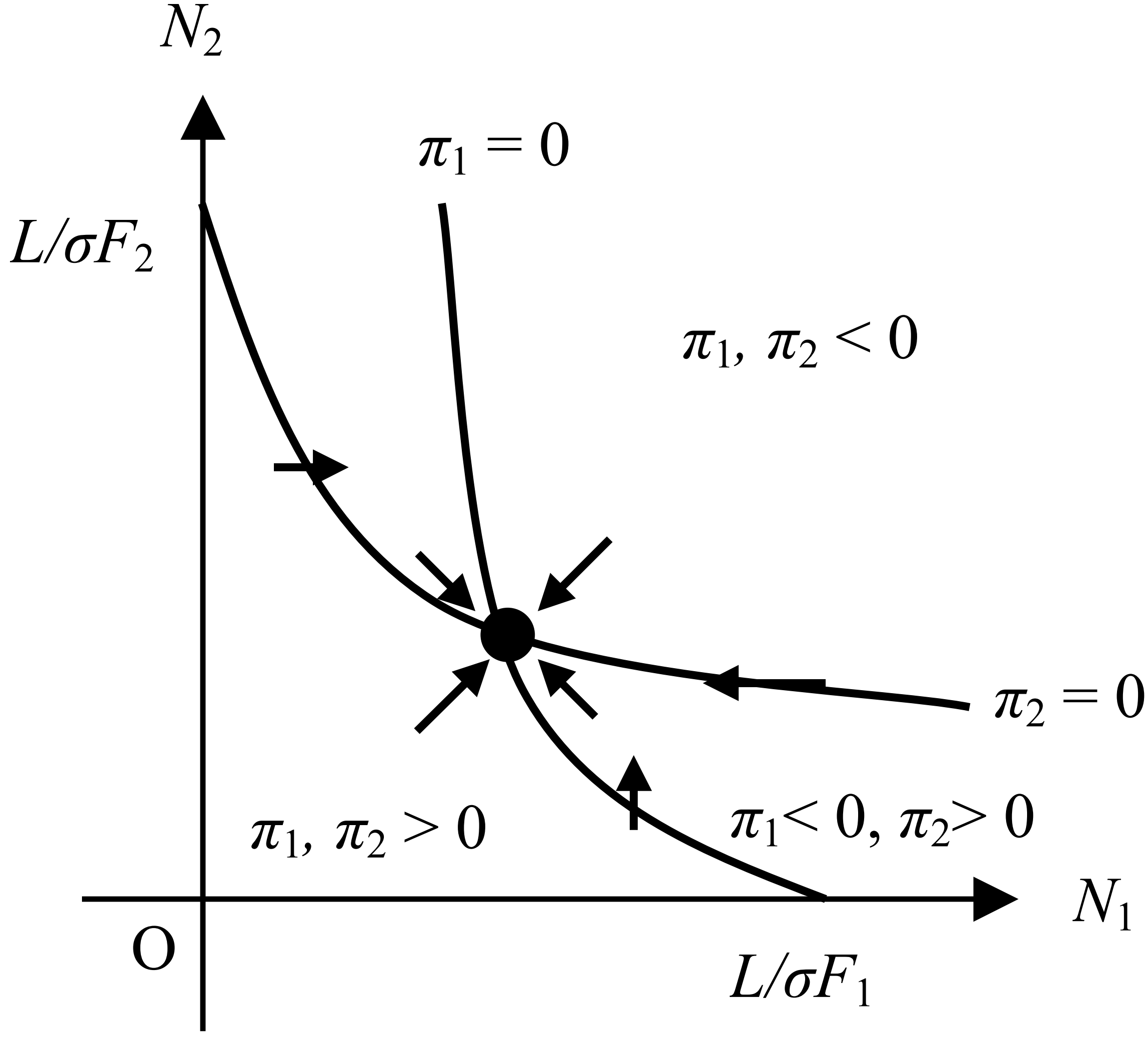

Figure 4

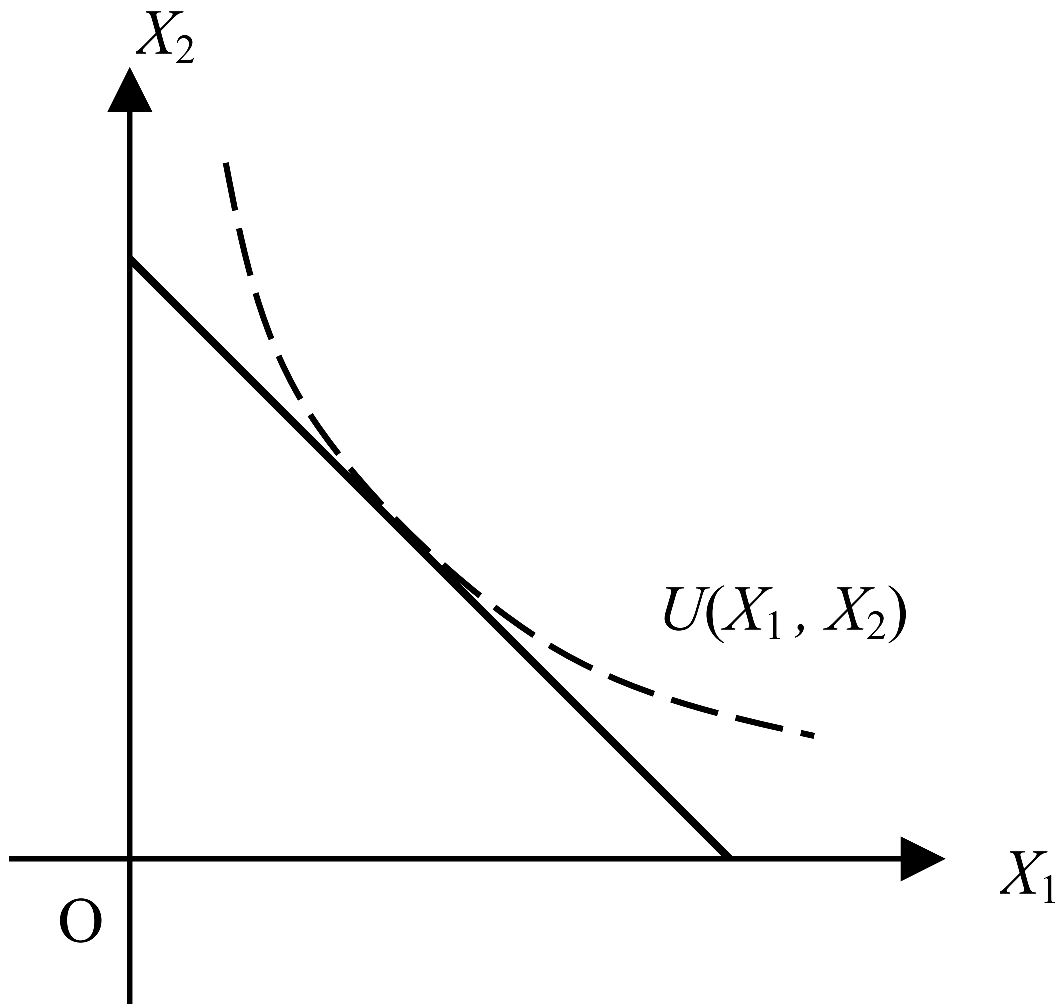

Figure 5

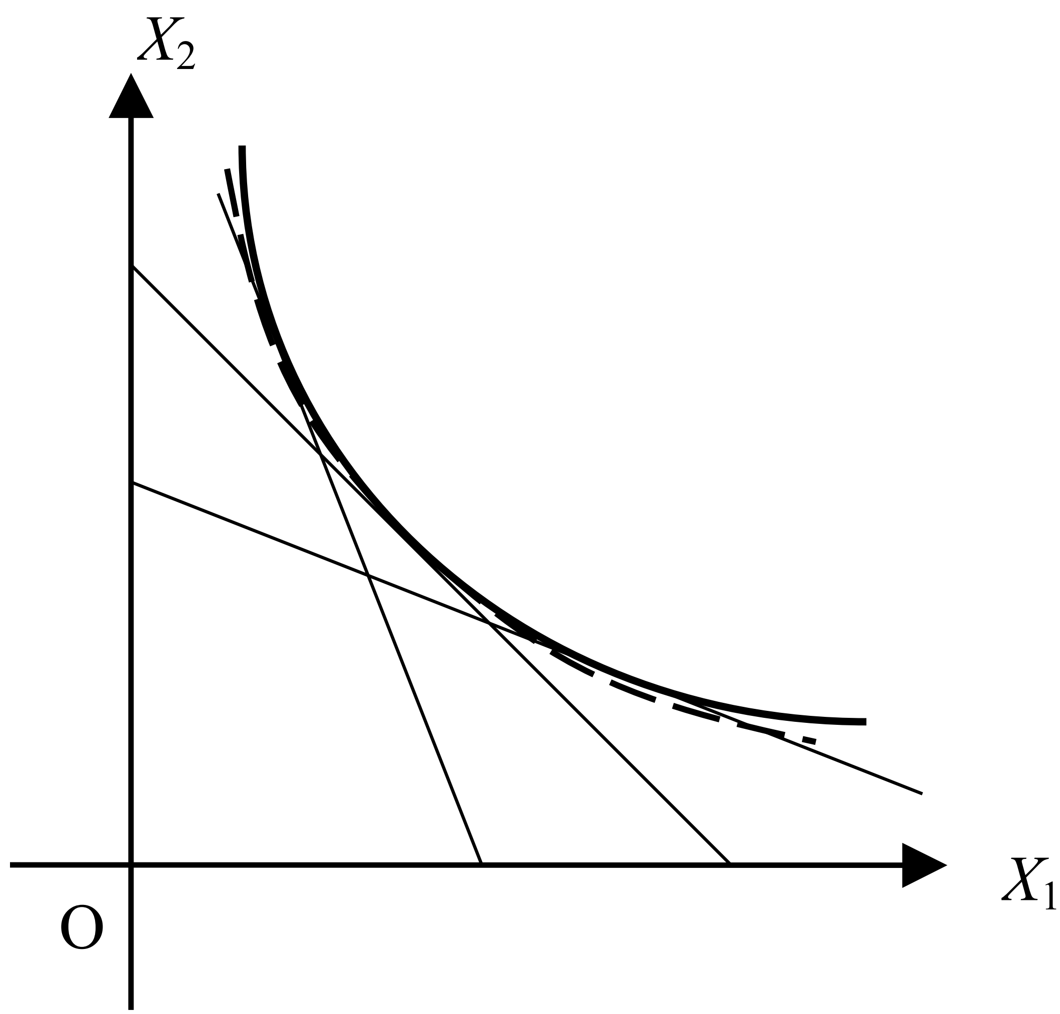

Figure 6

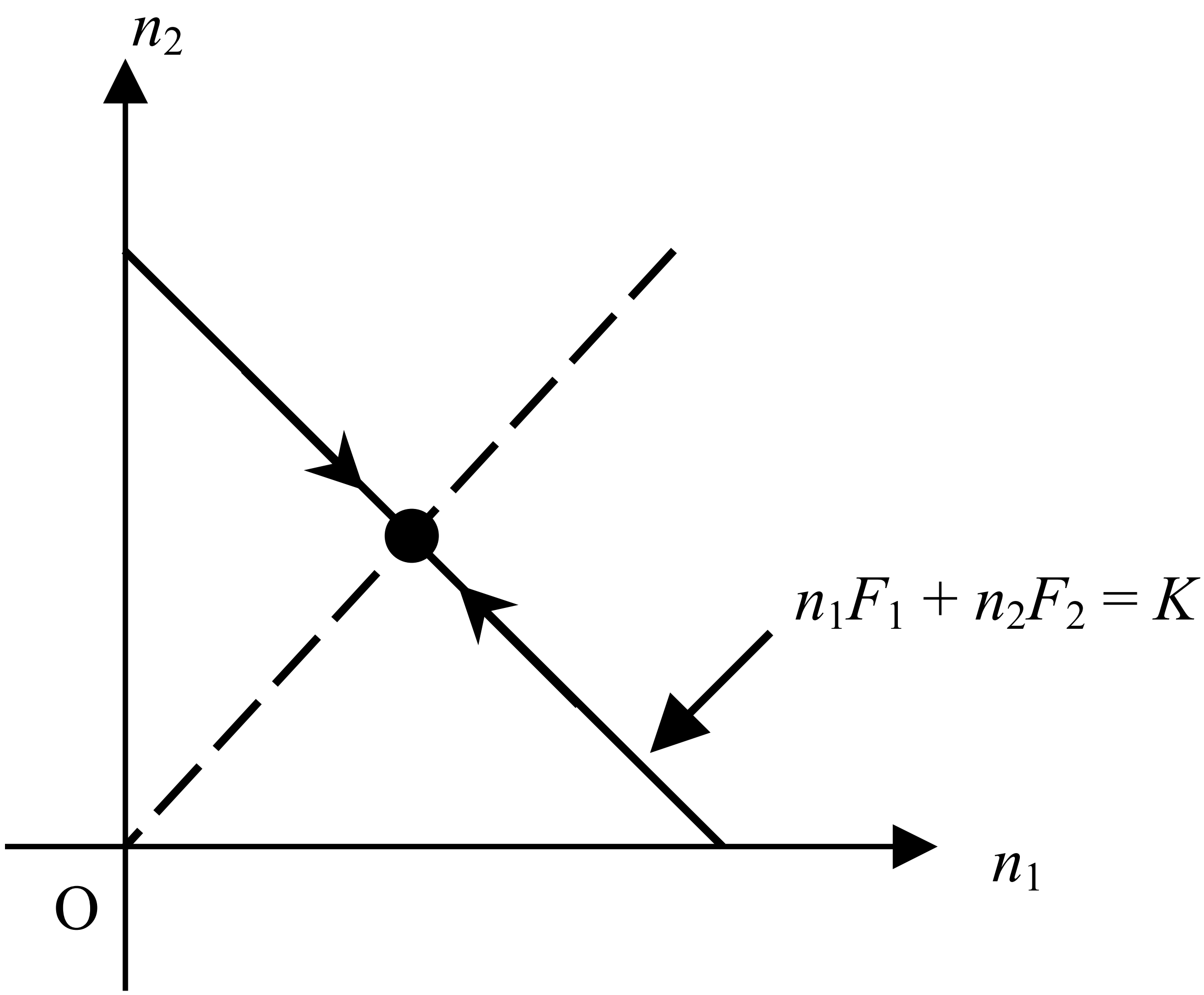

Figure 7