

# Foreclosing Competition through Access Charges and Price Discrimination<sup>1</sup>

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## **Abstract**

This article analyzes competition between two asymmetric networks, an incumbent and a new entrant. Networks compete in non-linear tariffs and may charge different prices for on-net and off-net calls. Departing from cost-based access pricing allows the incumbent to foreclose the market in a profitable way. If the incumbent benefits from customer inertia, then it has an incentive to insist in the highest possible access markup even if access charges are reciprocal and even in the absence of actual switching costs. If instead the entrant benefits from customer activism, then foreclosure is profitable only when switching costs are large enough.

# 1 Introduction

Telecommunication networks need access to rivals' customers in order to provide universal connectivity. This need for interconnection requires cooperation among network operators, who must agree on access conditions and, in particular, on termination charges (also called access charges). These wholesale arrangements affect the operators' cost of off-net calls and thus have an impact on retail competition among the operators. This raises two concerns. The first is that cooperation over interconnection may be used to soften downstream competition; the second is that established network operators may use access charges to foreclose the market.

The former issue was first addressed by Armstrong (1998) and Laffont, Rey and Tirole (1998a), who show that high access charges indeed undermine retail competition when networks compete in linear prices and do not price discriminate on the basis of where the call terminates.<sup>1</sup> Laffont, Rey and Tirole (1998a) show however that access charges lose their collusive power when networks compete in other dimensions, as is the case of two-part tariffs, due to a waterbed effect.<sup>2</sup> An increase in the access charge inflates usage prices, but this makes it more attractive to build market share, which results in fiercer competition for subscribers and lower fixed fees: networks can actually find it worthwhile to spend the full revenue from interconnection fees to build market share, so that termination charges no longer affect equilibrium profits. This profit neutrality has since been further studied and shown to depend on three assumptions: full-participation, no termination-based price discrimination and network symmetry.<sup>3</sup> López (2007) moreover extends the previous static analyses and shows that, in a dynamic setting, even symmetric networks with full consumer participation can use (future) reciprocal access charges to soften current competition.<sup>4</sup>

In the case of termination-based price discrimination, Gans and King (2001), building

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<sup>1</sup>High termination charges raise on average the marginal cost of calls, which encourages operators to maintain high prices.

<sup>2</sup>The term "waterbed effect" was first coined by Prof. Paul Geroski during the investigation of the impact of fixed-to-mobile termination charges on retail prices. See also Genakos and Valletti (2007).

<sup>3</sup>See Armstrong (2002) and Vogelsang (2003) for a survey of this literature.

<sup>4</sup>Since departing from cost-based termination charges adversely affects larger networks, this in turn reduces networks' incentives to build market shares.

on Laffont, Rey and Tirole (1998b), show that a (reciprocal) access charge below cost reduces competition. The intuition is that off-net calls being then cheaper than on-net calls, customers favour smaller networks; as a result, networks bid less aggressively for market share, which raises the equilibrium profits. However, in practice regulators are usually concerned that access charges are too high rather than too low, particularly for mobile operators. As stressed by Armstrong and Wright (2008), this may stem from the fact that “wholesale arbitrage” limits mobile operators’ ability to maintain high fixed-to-mobile (FTM) charges<sup>5</sup> alongside low mobile-to-mobile (MTM) charges, since fixed-line networks could “transit” their calls via another mobile operator in order to benefit from a lower MTM charge.<sup>6</sup>

The second traditional concern is that cooperation might be insufficient. This issue usually arises in markets where large incumbent operators face competition from smaller rivals, and may be tempted to degrade connectivity or use access charges to foreclose the market. Indeed, small mobile operators often complain that a high termination charge hurts their ability to compete in an effective way with large networks. Two arguments are normally used to motivate this concern. The first is a supply-side argument, whereby small operators face higher long-run incremental costs than larger operators due to scale economies.<sup>7</sup> European national regulatory agencies (NRAs) have for example relied on this argument to justify the adoption of asymmetric termination rates.<sup>8</sup>

The second argument, which is the focus of this paper, is the presence of demand-side network effects resulting from termination-based price discrimination. If for example

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<sup>5</sup>Historically, fixed and mobile operators were not really competing against each other, and thus a traditional “one-way access” analysis applied. Termination charges between those two types of networks are moreover usually asymmetric, different termination costs and regulatory constraints leading to relatively low charges for mobile-to-fixed calls and substantially higher charges for fixed-to-mobile calls.

<sup>6</sup>If mobile operators must adopt the same termination charge for FTM and MTM calls, this uniform charge may then be above cost if the waterbed effect on FTM is limited or if operators set their own charges unilaterally.

<sup>7</sup>It is also argued that cost differences may be exacerbated by staggered entry dates, unequal access to spectrum and (lack of) integration between fixed and mobile services.

<sup>8</sup>See for example the decision of the Belgian NRA (*Décision du Conseil de l’IBPT*) of 11 August 2006, the Decision 2007-0810 of October 4 2007 by the French NRA (ARCEP), the decision (Delibera 3/06/CONS) adopted by the Italian NRA (AGCOM) in January 2006 or the three decisions adopted by the Spanish NRA (CMT) on 28 September 2006 (Decisions AEM 2006/724, AEM 2006/725 and AEM 2006/726). See also the review of mobile call termination by the regulator and competition authority for the UK communications industries (OFCOM Mobile Call Termination Statement, 27 March 2007).

the termination charge is above cost, then prices will be lower for on-net calls; as a result, customers favour larger networks, in which a higher proportion of calls remain on-net. Some European NRAs have also relied on this demand-side argument to call for asymmetric termination charges. For example, in its Decision of October 2007, the French regulator stressed the presence of network effects due to off-net/on-net tariff differentials that impede smaller networks' ability to compete effectively.<sup>9</sup> Similarly, in its Decision of September 2006,<sup>10</sup> the Spanish regulator argued that network effects can place smaller networks at a disadvantage, and that higher access charges can increase the size of such network effects. And in the Common Position adopted on February 2008,<sup>11</sup> the European regulators express the concern that, because of network effects, "an on-net/off-net retail price differential, together with significantly above-cost mobile termination rates, can, in certain circumstances, tone down competition to the benefit of larger networks".<sup>12</sup>

To explore this issue, we analyze the competition between two asymmetric networks, an incumbent and a new entrant. Customers are initially attached to the incumbent network and incur switching costs if moving to the other network. Thus, as in Klemperer (1987), to build market share the entrant must bid more aggressively for customers than the incumbent, which therefore enjoys greater market power. In particular, the incumbent can keep monopolizing the market when switching costs are large enough; as we will see, when switching costs are not that large, departing from cost-based termination charges can help the incumbent maintain its monopoly position and profit.

We first consider the case where networks not only compete in subscription fees and in usage prices, but can moreover charge different prices for on-net and off-net calls. Such on-net pricing creates price-mediated network effects and, as a result, the incumbent can indeed keep the entrant out of the market and still charge monopoly prices by setting a large enough markup (or subsidy) on the access charge, even if access charges are

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<sup>9</sup>See section 4.2.2 of ARCEP Decision 2007-0810 of October 4 2007.

<sup>10</sup>Decision AEM 2006/726, p. 13, 14 and 33.

<sup>11</sup>See "ERG's Common Position on symmetry of fixed call termination rates and symmetry of mobile call termination rates", adopted by the ERG-Plenary on 28th February 2008, p. 96-102. Available at <http://www.erg.eu.int/>.

<sup>12</sup>The Common Position also stresses that these network effects can be exacerbated via incoming calls: a high off-net price will reduce the amount of off-net calls, which in turn lowers the value of belonging to the smaller network since less people will then call the customers of that network.

reciprocal. If the incumbent benefits from customer inertia,<sup>13</sup> then it has actually an incentive to insist on the highest possible (reciprocal) access markup, so as to foreclose the market and exploit fully the resulting monopoly power. Customer inertia thus provides a form of "virtual" switching costs which, combined with high termination charges, is a good substitute for "real" switching costs: in the presence of customer inertia, the incumbent can corner the market and earn the monopoly profit even in the absence of any real switching costs. A large termination subsidy could also yield the same outcome; however feasibility constraints may limit subsidies, which may moreover trigger various types of arbitrage. The scope for foreclosure is more limited when the entrant benefits from customer activism; while the incumbent may still try to prevent entry, too high an access charge would allow the entrant to overtake the incumbent. The incumbent may then prefer to set an above- or below-cost access charge, and foreclosure strategies are profitable only when switching costs are sufficiently large.

Our analysis also extends the insight of Gans and King (2001) and shows that, as long as the two networks share the market, a small access subsidy generates higher equilibrium profits (for both networks) than any positive access markup. Yet, it does not follow that both networks will agree to subsidizing access, since a large enough access markup may instead allow the incumbent to corner the market, and higher levels might moreover allow the incumbent to earn the full monopoly profit. Our analysis thus supports the conventional wisdom that well-established networks prefer high access charges, and seems to call for regulatory authorities to set bounds on access markups (and subsidies).

Finally, we show that termination-based price discrimination is a key factor. Indeed, absent on-net pricing, foreclosure strategies are never profitable – and moreover no longer feasible in a receiver pays regime.

There are only few insights from the academic literature on the impact of mobile operators' termination rates on entry or predation. Calzada and Valletti (2008) extend Gans and King's analysis to a (symmetric) multi-firm industry; they stress that incumbents

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<sup>13</sup>Since on-net pricing generates club effects, consumers face coordination problems and there may exist multiple consumer responses to a given set of prices. "Customer inertia" refers to the situation where, in case of multiple responses, consumers adopt the response that is favourable to the incumbent.

may favour above-cost termination charges when new operators face entry costs: for any given number of firms, increasing the charge above cost decreases the equilibrium profits but, by the same token, limits the number of entrants; overall, this allows incumbent operators to increase their own profits. Hoernig (2007) analyzes predatory pricing in the presence of call externalities (i.e., taking into account the utility of receiving calls) and termination-based price discrimination, for given termination charges. He shows that call externalities give the incumbent an incentive to increase its off-net price in order to make a smaller rival less attractive (as it will receive fewer or shorter calls), and this incentive is even higher when the incumbent engages in predatory pricing and seeks to reduce its rival's profit. Both papers thus study how incumbents can reduce rivals' profitability in order to limit entry, at the expense of a (possibly temporary) loss in its own profit. In contrast, we study how the incumbent can manipulate the termination charge (even when it is reciprocal) to *increase* its own profit at the expense of the entrant.

The article is organized as follows. Section 2 describes the model. Section 3 analyses retail competition for a given, reciprocal, access charge. It first characterizes shared-market equilibria and extends the insight of Gans and King to asymmetric networks; it then studies under what conditions one network may corner the market. Section 4 draws the implications for the determination of the access charge and shows that, despite Gans and King's insight, an incumbent network may favour a high access charge in order to foreclose the market. Section 5 analyses the case of no termination-based price discrimination, while Section 6 considers a receiver pays regime. Section 7 concludes.

## 2 The model

Except for the existence of switching costs the setup is basically the same as in Laffont, Rey and Tirole (1998b). There are two networks: an incumbent,  $I$ , and an entrant,  $E$ . Both networks have the same cost structure. It costs  $f$  to connect a customer, and each call costs  $c \equiv c_O + c_T$ , where  $c_O$  and  $c_T$  respectively denote the costs borne by the originating and terminating networks. To terminate an off-net call, the originating

network must pay a reciprocal access charge  $a$  to the terminating network. The access markup is thus equal to:

$$m \equiv a - c_T.$$

Networks offer substitutable services but are differentiated à la Hotelling. Consumers are uniformly distributed on the segment  $[0, 1]$ , whereas the two networks are located at the two ends of this segment. Consumers' tastes are represented by their position on the segment and taken into account through a "transportation" cost  $t > 0$ , which reflects their disutility from not enjoying their ideal type of service. For a given volume of calls  $q$ , a consumer located at  $x$  and joining network  $i = I, E$  located at  $x_i \in \{0, 1\}$  obtains a gross utility given by:

$$u(q) - t|x - x_i|,$$

where  $u(q)$  denotes the variable gross surplus, with  $u' > 0 > u''$  and  $u'(0) < +\infty$ . To ensure full participation we will assume throughout the paper that the surplus derived from being connected to either network is sufficiently large:  $u(0) \gg t$ . In addition, consumers switching to  $E$ 's network incur a cost  $s > 0$ .

Each network  $i = I, E$  offers a three-part tariff:

$$T_i(q, \hat{q}) = F_i + p_i q + \hat{p}_i \hat{q},$$

where  $F_i$  is the fixed subscription fee and  $p_i$  and  $\hat{p}_i$  respectively denote the on-net and off-net usage prices.

Let  $\alpha_i$  denote network  $i$ 's market share. Assuming a balanced calling pattern,<sup>14</sup> the net surplus offered by network  $i$  is (for  $i \neq j = I, E$ ):

$$w_i = \alpha_i v(p_i) + \alpha_j v(\hat{p}_i) - F_i, \tag{1}$$

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<sup>14</sup>This assumption implies that the proportions of calls originating on a given network and completed on the same or the other network reflect networks' market shares.



where

$$v(p) \equiv \max_q u(q) - pq$$

denotes the consumer surplus for a price  $p$ .

In a first step, we will take as given the reciprocal termination charge and study the subsequent competition game where the networks set simultaneously their retail tariffs (subscription fees and usage prices), and then consumers choose which network to subscribe and how much to call. In a second step we discuss the determination of the termination charge. Before that, we characterize the consumer response to networks' prices and provide a partial characterization of the equilibrium prices.

**Marginal cost pricing.** As usual, networks find it optimal to adopt cost-based usage prices. Network  $i$ 's profit is equal to:

$$\pi_i \equiv \alpha_i [\alpha_i(p_i - c)q(p_i) + \alpha_j(\hat{p}_i - c - m)q(\hat{p}_i) + F_i - f] + \alpha_i\alpha_jmq(\hat{p}_j). \quad (2)$$

Adjusting  $F_i$  so as to maintain net surpluses  $w_I$  and  $w_E$  and thus market shares constant,<sup>15</sup> then leads network  $i$  to set its prices  $p_i$  and  $\hat{p}_i$  so as to maximize

$$\alpha_i \{ \alpha_i [(p_i - c)q(p_i) + v(p_i)] + \alpha_j [(\hat{p}_i - c - m)q(\hat{p}_i) + v(\hat{p}_i)] - w_i - f \} + \alpha_i\alpha_jmq(\hat{p}_j),$$

which yields marginal-cost pricing:

$$p_i = c, \hat{p}_i = c + m.$$

Thus, both networks always charge usage prices that reflect the perceived cost of calls: the true cost  $c$  for on-net calls, augmented by the access markup  $m$  for off-net calls. As a result, while each network  $i$  must pay  $\alpha_i\alpha_jmq(\hat{p}_i)$  to its rival, there is no net interconnection payment; since both networks charge the same off-net price ( $\hat{p}_i = \hat{p}_j = c + m$ ), neither the incumbent nor the entrant has a net outflow of calls:  $\alpha_i\alpha_jm(q(\hat{p}_j) - q(\hat{p}_i)) = 0$ ,

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<sup>15</sup>As already noted, on-net pricing can generate multiple consumer responses to a given set of prices. We assume here that changing tariffs so as to keep net surpluses constant does not trigger consumers to switch to alternative responses, if they exist.

whatever the networks' market shares.

**Network Externalities and market shares.** Since the off-net price increases with the access markup, departing from cost-based termination charges generates tariff-mediated network externalities. For example, if the access markup is positive, prices are higher for off-net calls and the subscribers of a given network are thus better off, the more customers join that network. As a result, there may exist multiple consumer responses to the same set of prices.

We now determine the consumer responses to given subscription fees  $F_I$  and  $F_E$ , together with cost-based usage prices. If consumers anticipate market shares  $\bar{\alpha}_I$  and  $\bar{\alpha}_E = 1 - \bar{\alpha}_I$ , then they expect a net surplus

$$w_i = \bar{\alpha}_i v(c) + \bar{\alpha}_j v(c + m) - F_i. \quad (3)$$

from subscribing to network  $i$ , for  $i \neq j = I, E$ . A consumer located at a distance  $x \in [0, 1]$  from network  $I$  is therefore willing to stay with that network when  $w_I - tx \geq w_E - t(1 - x) - s$  and prefers to switch otherwise. In a shared-market outcome, the actual consumer response,  $\hat{\alpha}_i$ , as a function of consumers' expectation  $\bar{\alpha}_i$ , is therefore given by

$$\begin{aligned} \hat{\alpha}_i(\bar{\alpha}_i) &= \frac{1}{2} + \sigma(w_i - w_j + \delta_i s) \\ &= \frac{1}{2} + \sigma(F_j - F_i + \delta_i s) + 2\sigma\left(\bar{\alpha}_i - \frac{1}{2}\right)(v(c) - v(c + m)), \end{aligned} \quad (4)$$

where  $\delta_I \equiv 1$ ,  $\delta_E \equiv -1$ , and  $\sigma \equiv 1/2t$  measures the substitutability between the two networks.

Any fixed point  $\bar{\alpha}_i = \hat{\alpha}_i(\bar{\alpha}_i)$  that lies in  $(0, 1)$  constitutes a consumer response where the networks share the market; combining (3) and (4) then yields network  $i$ 's market share, as a function of both subscription fees:

$$\alpha_I = 1 - \alpha_E = \frac{1}{2} + \frac{F_E - F_I + s}{2\tau(m)}, \quad (5)$$

where

$$\tau(m) \equiv t - (v(c) - v(c + m)).$$

Similarly, there exists a continuation equilibrium where network  $i$  corners the market if  $\hat{\alpha}_i(1) \geq 1$ , and a continuation equilibrium where network  $j$  corners the market if  $\hat{\alpha}_i(0) \leq 0$ .

Note that the function  $\hat{\alpha}_i$  has a constant slope, equal to

$$\frac{d\hat{\alpha}_i}{d\bar{\alpha}_i} = \frac{v(c) - v(c + m)}{t}.$$

It follows that when

$$\tau(m) > 0 \tag{6}$$

the slope  $d\hat{\alpha}_i/d\bar{\alpha}_i$  is always lower than 1 (and is even negative for  $m < 0$ ), which in turn implies that there exists a unique consumer response (see Figure 1, which plots the "reaction to anticipations"  $\max\{0, \min\{1, \hat{\alpha}(\cdot)\}\}$ ).

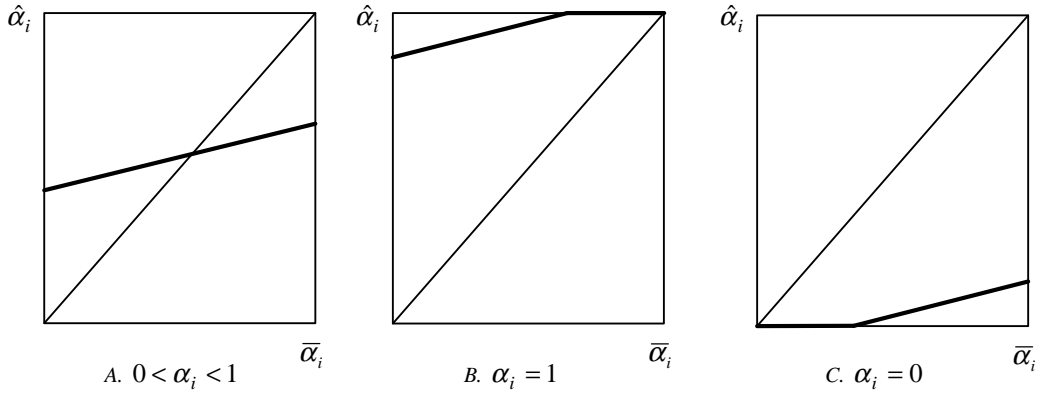


Figure 1: Unique and stable consumer response:  $v(c) - v(c + m) < t$ .

Condition (6) depends only on the termination markup and on the transportation parameter, and not on the fixed fees  $F_I$  and  $F_E$  or the market shares; it is moreover strictly satisfied for any  $m \leq 0$ . Therefore, when  $m \leq 0$  or  $m > 0$  but not too large, for any given fixed fees  $F_I$  and  $F_E$ , there exists a unique consumer response, which can be characterized as follows. When the expression in (5) lies in  $(0, 1)$ , the two networks share

the market and network  $i$ 's market share,  $\alpha_i(F_I, F_E)$ , is precisely given by (5) (see Figure 1.A). When instead this expression exceeds 1, network  $i$  corners the market (Figure 1.B); finally, when this expression is negative, the other network corners the market (Figure 1.C).

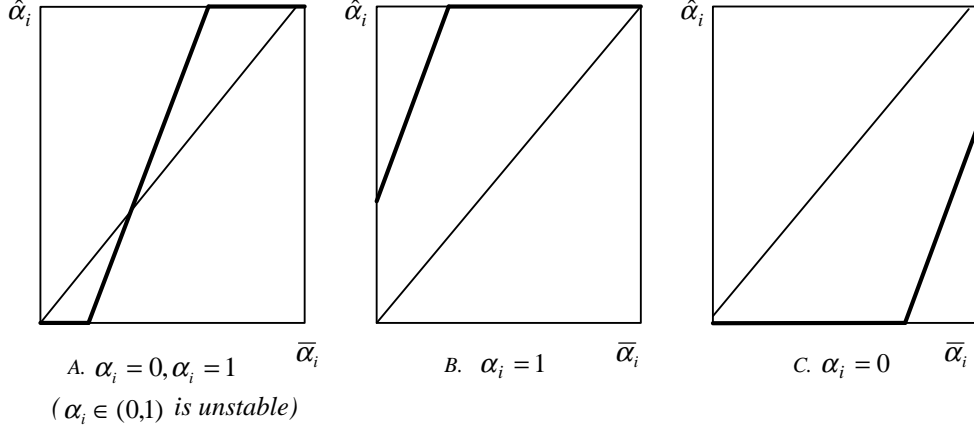


Figure 2: Cornered-market stable consumer responses:  $v(c) - v(c + m) > t$ .

Condition (6) may not hold, however, when  $m$  is positive and large. There may then exist multiple consumer responses, as illustrated in Figure 2.A, where three possible consumer responses exist: two cornered-market outcomes and one shared-market outcome. The shared-market outcome is however unstable: a small increase in the market share of any network triggers a cumulative process in favour of that network, and this process converges towards that network cornering the market. In contrast, the two cornered-market outcomes are stable. In particular, starting from a situation where all consumers are with the incumbent, a few customers making a "mistake" and switching to the entrant would not trigger any snowballing in favour of the entrant; the customers would thus regret their mistake and wish to have stayed with the incumbent. Since customer inertia may favour the incumbent, in the case of multiple consumer responses it may be reasonable to assume that the stable outcome where consumers stick to the incumbent network is the most plausible outcome. Yet, throughout the paper, we will also take into consideration the possibility of alternative consumer responses and study under what conditions the incumbent can make sure to keep the rival out of the market.

### 3 Price competition

We now characterize the equilibrium fixed fees, given the consumer response determined in the previous section.

#### Shared-market equilibria

In the light of the above analysis, a price equilibrium yielding a stable shared-market outcome can exist only when (6) holds, in which case the consumer response is moreover always unique. We denote by  $\alpha_i(F_I, F_E)$  the corresponding market share of network  $i = I, E$ . Since usage prices reflect costs, network  $i$ 's profit can be written as (for  $i \neq j = I, E$ ):

$$\pi_i = \alpha_i(F_I, F_E) [F_i - f + \alpha_j(F_I, F_E) mq(c + m)]. \quad (7)$$

**Best responses.** Given the rival's fee  $F_j$ , we can use the market share definition (5) to express  $F_i$  and  $\pi_i$  as a function of  $\alpha_i$ :

$$\begin{aligned} F_i &= F_j + \tau(m) + \delta_i s - 2\tau(m) \alpha_i, \\ \pi_i(\alpha_i) &= \alpha_i [F_j + \tau(m) + \delta_i s - f + mq(c + m) - 2\varphi(m) \alpha_i], \end{aligned} \quad (8)$$

where

$$\varphi(m) \equiv \tau(m) + \frac{mq(c + m)}{2},$$

and  $\delta_I = -\delta_E = 1$ . The first-order derivative is

$$\frac{d\pi_i}{d\alpha_i} = F_j + \tau(m) + mq(c + m) + \delta_i s - f - 4\varphi(m) \alpha_i, \quad (9)$$

while the second-order derivative is negative if and only if:

$$\varphi(m) > 0. \quad (10)$$

When this second-order condition holds, we have:

- if  $F_j + \tau(m) + mq(c + m) + \delta_i s - f \leq 0$ , network  $i$ 's best response is to leave the market to its rival (i.e.,  $\alpha_i = 0$ ), and any  $F_i^r \geq F_j + \delta_i s + \tau(m)$  is thus a best-response to  $F_j$  (see the dashed areas in Figure 3);
- if  $F_j + \tau(m) + mq(c + m) + \delta_i s - f \geq 4\varphi(m)$ , network  $i$ 's best response is to corner the market ( $\alpha_i = 1$ ), and thus  $F_i^r(F_j) = F_j + \delta_i s - \tau(m)$  ( $45^\circ$  lines in Figure 3),
- if  $4\varphi(m) > F_j + \tau(m) + mq(c + m) + \delta_i s - f > 0$ , network  $i$ 's best response entails a shared-market outcome,  $\alpha_i \in (0, 1)$ :

$$\alpha_i = \frac{F_j + \tau(m) + mq(c + m) + \delta_i s - f}{4\varphi(m)}, \quad (11)$$

that is, network  $i$ 's best response is given by (middle zone in Figure 3):

$$F_i^r = \frac{(\tau(m) + mq(c + m))(F_j + \delta_i s) + \tau(m)(f + \tau(m))}{2\varphi(m)}.$$

where the denominator is positive as long as the second-order condition holds.

**Equilibrium.** Solving for the first-order conditions yields:

$$F_i = f + \tau(m) + \frac{\tau(m) + mq(c + m)}{3\psi(m)}\delta_i s, \quad (12)$$

where

$$\psi(m) \equiv \tau(m) + \frac{2}{3}mq(c + m).$$

Substituting (12) into (5), equilibrium market shares are given by

$$\alpha_I = 1 - \alpha_E = \frac{1}{2} \left( 1 + \frac{s}{3\psi(m)} \right). \quad (13)$$

It is easy to check that  $\psi(m) > 0$  in any candidate shared-market equilibrium,<sup>16</sup> which implies that the market share  $\alpha_I$  exceeds 1/2 and increases with  $s$ . Therefore, it cor-

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<sup>16</sup>When subscription fees are (weak) strategic complements ( $\partial F_i / \partial F_j \geq 0$ , or  $\tau(m) + mq(c + m) \geq 0$ ), (6) implies  $\psi(m) > 0$ , since  $3\psi(m) = 2(\tau(m) + mq(c + m)) + \tau(m) > 0$ ; when subscription fees are instead strategic substitutes ( $\partial F_i / \partial F_j < 0$ , or  $\tau(m) + mq(m) < 0$ ), the candidate equilibrium is stable (i.e.,  $\partial F_i / \partial F_j > -1$ ) if and only if  $\psi(m) > 0$ .

responds indeed to a shared-market equilibrium (i.e.,  $\alpha_i < 1$ ) when and only when  $s$  is small enough, namely, when

$$\psi(m) > \frac{s}{3}. \quad (14)$$

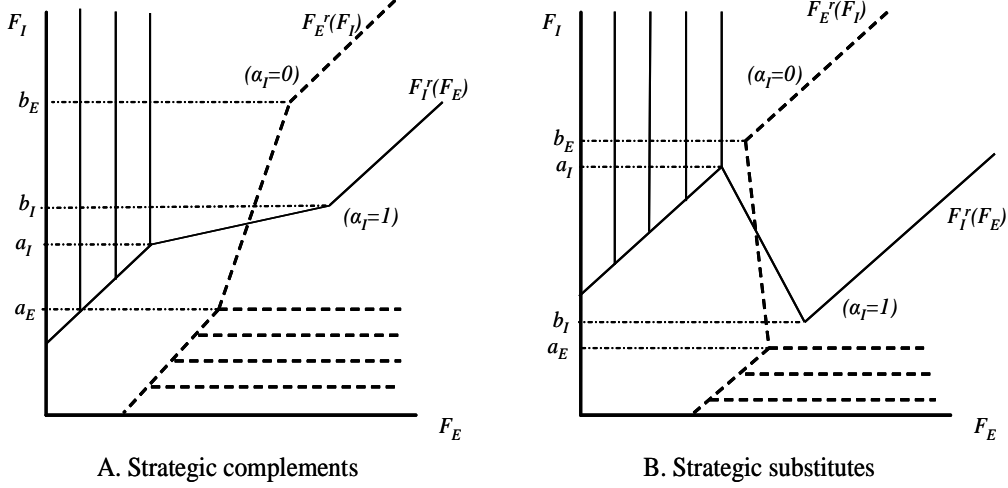


Figure 3: Shared-market equilibria.  $a_I = f - mq(c + m)$ ,  $b_I = f + 2\tau(m) + mq(c + m)$ ,  $a_E = f + s - \tau(m) - mq(c + m)$ ,  $b_E = f + s + 3\tau(m) + mq(c + m)$ .

When  $m \geq 0$ , (6) implies (10) and  $0 < \partial F_i / \partial F_j < 1$ . When instead  $m < 0$ , (6) is always satisfied and subscription fees remain strategic complements (i.e.,  $\partial F_i / \partial F_j > 0$ ) as long as  $\tau(m) + mq(c + m) > 0$ , in which case (10) also holds and  $\partial F_i / \partial F_j < 1$ . Therefore, in those two situations, whenever the shared-market condition (14) holds there exists a unique price equilibrium, as illustrated by Figure 3.A; this equilibrium involves a shared market characterized by (12), strategic complementarity and stability. If instead  $m < 0$  and  $\tau(m) + mq(c + m) < 0$ , subscription fees are strategic substitutes. However, the shared-market condition (14) then implies (10) and  $\partial F_i / \partial F_j > -1$ ; therefore, the price equilibrium is again unique and stable, as illustrated by Figure 3.B, and involves again a shared market characterized by (12). In all cases, (6) moreover implies that consumer responses to prices yield a stable market outcome. Thus, we have:

**Proposition 1** *A stable price equilibrium yielding a stable shared-market outcome exists, in which case it is the unique price equilibrium, if and only if (6) and (14) hold.*

Proposition 1 shows that a stable shared-market equilibrium exists when the termination charge is not too high (condition (6)) and switching costs are moreover moderate (condition (14)). For example, for cost-based access charges ( $m = 0$ ), such an equilibrium exists when  $s < 3t$ .<sup>17</sup> When this condition is satisfied, a shared-market equilibrium also exists (and is then the unique equilibrium) when the termination markup is positive, as long as (6) and (14) remain satisfied.

**Comparative statics.** We now study the impact of the access charge on shared-market equilibrium profits. Gans and King (2001) show that symmetric networks prefer access charges below marginal costs. Intuitively, when  $m$  is negative, off-net calls are priced below on-net calls, so consumers prefer to join smaller networks, all else being equal. Consequently, networks bid less aggressively for marginal customers. The next proposition confirms that, as long as the two networks share the market, price competition is softened when  $m$  decreases below zero, independently of networks' sizes.

**Proposition 2** *In the range of termination charges yielding a shared-market equilibrium, there exists a termination subsidy ( $m < 0$ ) that gives both networks greater profits than any non-negative termination markup.*

**Proof.** See Appendix. ■

Proposition 2 extends the insight of Gans and King to asymmetric networks. It however only applies to termination markups that are small enough to yield a shared-market equilibrium. As we will see, networks may actually favour more extreme termination markups that allow them to corner the market and charge high prices.<sup>18</sup>

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<sup>17</sup>As mentioned earlier, to ensure full participation we assume throughout the analysis that  $t$  is small enough, compared with the utility derived from being connected to either network. Under cost-based access charges, the marginal consumer's net utility is equal to:

$$v(c) - F_I - t\alpha_I = v(c) - f - \frac{3t + s}{2}.$$

Therefore, a sufficient condition for full participation is  $v(c) - f > 3t$ , since then the marginal consumer obtains a positive net utility whenever a shared-market equilibrium exists, i.e., whenever  $s < 3t$ .

<sup>18</sup>The same comment applies to the case of symmetric operators considered by Gans and King (which corresponds here to  $s = 0$ ). While they show that networks' symmetric shared-market equilibrium profits are maximal for a negative mark-up, more extreme mark-ups (including positive ones) may induce cornered-market equilibria that generate greater industry profits.



## Cornered-market equilibria

We now study under what conditions a network operator can corner the market.

Suppose first that (6) still holds, ensuring that there is a unique consumer response to subscription fees. From the above analysis, a cornered-market equilibrium can then exist only when condition (14) fails to hold.

In a candidate equilibrium where network  $i$  corners the market, the consumers located at the other end of the segment must prefer to stick to  $i$ 's network; that is, for  $i \neq j = I, E$ :

$$v(c) - t - F_i \geq v(c + m) - \delta_i s - F_j,$$

or:

$$F_i \leq F_j - \tau(m) + \delta_i s. \quad (15)$$

Furthermore, if this inequality holds strictly then  $i$  can increase its subscription fee and still corner the market. Therefore, a necessary equilibrium condition is:

$$F_i = F_j - \tau(m) + \delta_i s. \quad (16)$$

In addition: (i) network  $i$  should not prefer to charge a higher fee and increase its margin at the expense of its market share; and (ii) its rival should not be able to attract consumers and make positive profits. The precise interpretation of these two conditions depends on the concavity of the profit functions.

**Concave profits.** When (10) also holds, each operator's profit is globally concave with respect to its own price; the relevant deviations thus involve marginal price changes leading to a shared-market outcome. A candidate equilibrium satisfying (16) is therefore indeed an equilibrium if and only if:

- Network  $i$  does not gain from a marginal increase in its fee;<sup>19</sup> given the previous analysis of best responses, this amounts to  $F_j + \tau(m) + \delta_i s - f + mq(c + m) \geq$

---

<sup>19</sup>Note that this condition ensures that  $i$  obtains a non-negative profit – otherwise, a small increase in  $F_i$  would reduce its loss. Indeed, (16) and (17) imply  $F_i > f$  when the second-order condition (10) holds.

$4(\tau(m) + mq(c + m)/2)$ , or:

$$F_j \geq f + 3\tau(m) + mq(c + m) - \delta_i s, \quad (17)$$

- The rival network  $j$  does not gain from a marginal reduction in its fee or, equivalently, cannot make a positive profit by attracting its closest consumers; this amounts to:

$$F_j \leq f - mq(c + m). \quad (18)$$

Network  $j$ 's fee must therefore lie in the range

$$f - mq(c + m) \geq F_j \geq f + 3\tau(m) + mq(c + m) - \delta_i s, \quad (19)$$

which is feasible only when

$$\psi(m) \leq \frac{\delta_i s}{3}. \quad (20)$$

For the incumbent ( $i = I$ , for which  $\delta_I = 1$ ), this condition is satisfied whenever (14) fails to hold. Any pair of subscription fees  $(F_I, F_E)$  satisfying

$$F_I = F_E - \tau(m) + s \quad (21)$$

and

$$f - mq(c + m) \geq F_E \geq f + 3\tau(m) + mq(c + m) - s \quad (22)$$

then constitutes a price equilibrium where  $I$  corners the market. Among those equilibria, only one does not rely on weakly dominated strategies for  $E$ , and is therefore trembling-hand perfect: this is the one where

$$F_E = f - mq(c + m), F_I = f + s - \tau(m) - mq(c + m).$$

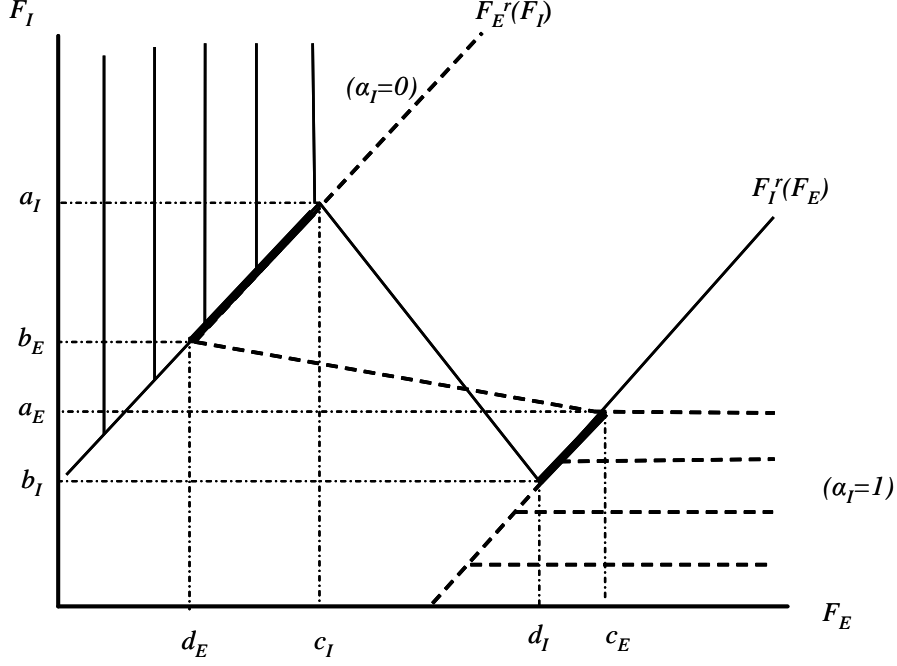


Figure 4: Cornered-market equilibria.  $a_I = f - mq(c + m)$ ,  $b_I = f + 2\tau(m) + mq(c + m)$ ,  $a_E = f + s - \tau(m) - mq(c + m)$ ,  $b_E = f + s + 3\tau(m) + mq(c + m)$ ,  $c_I = f - s - \tau(m) - mq(c + m)$ ,  $d_I = f - s + 3\tau(m) + mq(c + m)$ ,  $c_E = f - mq(c + m)$ ,  $d_E = f + 2\tau(m) + mq(c + m)$ .

By contrast,  $E$  can corner the market only if

$$\psi(m) \leq -\frac{s}{3}. \quad (23)$$

It follows that  $E$  cannot corner the market if  $m \geq 0$  (since the left-hand side is then positive under (6)); however, the left-hand side may become negative and possibly lower than  $-s/3$  when  $m$  is largely negative, in which case there can be a continuum of equilibria in which  $E$  corners the market by charging

$$F_E = F_I - \tau(m) - s, \quad (24)$$

including a unique trembling-hand perfect equilibrium where  $I$  sets  $F_I = f - mq(c + m)$  and  $E$  thus charges  $F_E = f - \tau(m) - mq(c + m) - s (> f)$ .

Note finally that, since (20) is more demanding for  $E$  than for  $I$ ,  $I$  can corner the market whenever  $E$  can do so (that is, both cornered market equilibria exist whenever  $E$

can corner the market). Figure 4 illustrates this case.

**Convex profits.** When (10) fails to hold, each operator's profit is convex with respect to its own subscription fee. The relevant strategies then consist in either cornering the market or leaving it to the rival. Thus, in a candidate equilibrium where  $I$  corners the market, it must be the case that:

- $I$  does not gain from "opting out", i.e., it should obtain a non-negative profit:

$$F_I \geq f.$$

- $E$  does not gain from lowering its subscription fee so as to corner the market, i.e., from charging  $F_E$  satisfying (24):

$$F_E = F_I - \tau(m) - s \leq f.$$

It follows that  $I$ 's equilibrium price must satisfy:

$$f + \tau(m) + s \geq F_I \geq f, \tag{25}$$

where the left-hand side is indeed always higher than the right-hand side under (6). Conversely, any set of prices satisfying (21) and (25) constitutes an equilibrium in which  $I$  corners the market.

We can similarly study under what conditions  $E$  can corner the market: condition (24) must hold,  $E$  must obtain a non-negative profit (i.e.,  $F_E \geq f$ ) and  $I$  should not be able to make a profit by cornering the market, i.e.:

$$F_I = F_E - \tau(m) + s \leq f.$$

Thus, in this equilibrium  $E$ 's equilibrium fee satisfies:

$$f + \tau(m) - s \geq F_E \geq f,$$

and such an equilibrium thus exists if and only if

$$s \leq \tau(m).$$

It follows that when  $E$  corners the market,  $I$ 's equilibrium price lies in the range  $[f + \tau(m) + s, f + 2\tau(m)]$ .

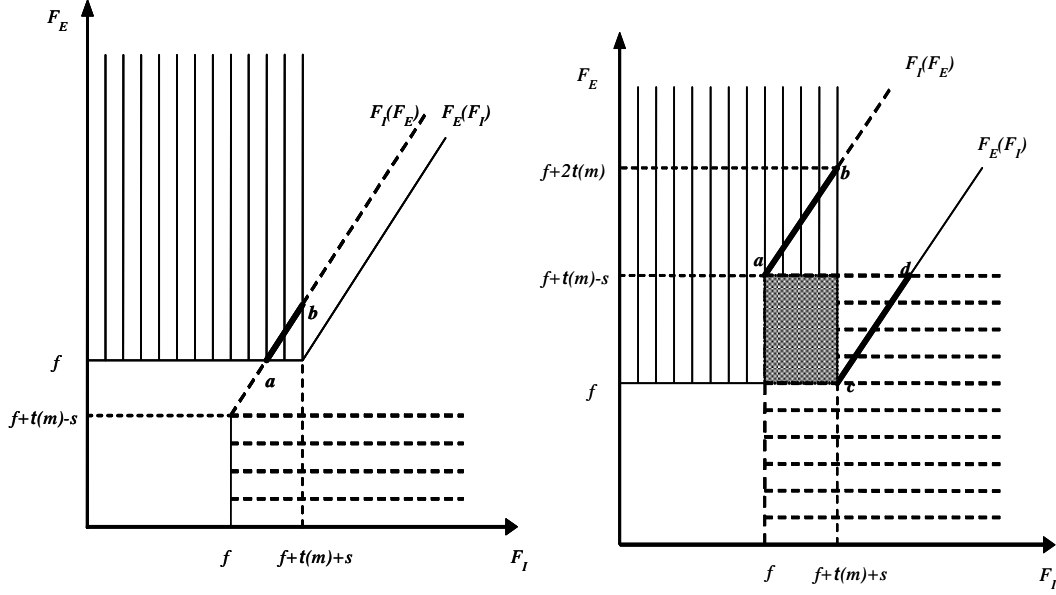


Figure 5: A) Only the incumbent corners the market:  $s > \tau(m)$ . B) The incumbent or the entrant corners the market:  $s \leq \tau(m)$ .

Figure 5 summarizes this analysis. When  $s > \tau(m)$ , only  $I$  can corner the market and it can achieve that while charging any price between  $f$  and  $f + \tau(m) + s$ . When instead  $s \leq \tau(m)$ , however,  $E$  may also corner the market.

**Multiple consumer responses.** Last, we turn to the case where (6) does not hold (i.e.  $\tau(m) \leq 0$ ), in which case there is never a stable shared-market consumer allocation, and there may be multiple cornered-market equilibria:

- when

$$F_E > F_I - \tau(m) - s, \tag{26}$$

there is a unique consumer response, in which  $I$  corners the market ( $\hat{\alpha}_i(0) > 0$ , Figure 2.B);

- when instead

$$F_I - \tau(m) - s \geq F_E \geq F_I + \tau(m) - s, \quad (27)$$

there are two stable consumer responses, in which either  $I$  or  $E$  corners the market ( $\hat{\alpha}_i(0) < 0$  and  $\hat{\alpha}_i(1) > 1$ , Figure 2.A);<sup>20</sup>

- finally, when

$$F_E < F_I + \tau(m) - s, \quad (28)$$

there is again a unique consumer response, in which  $E$  corners the market ( $\hat{\alpha}_i(1) < 1$ , Figure 2.C).

Obviously, a network can corner the market more easily when consumers favour that network in case of multiple responses to prices.

Suppose first that customer inertia, say, systematically favours the incumbent in the "middle" case corresponding to (27). Then  $I$  wins the whole market as long as  $F_I - F_E \leq s - \tau(m)$ , otherwise  $E$  wins the market. Since  $s - \tau(m) > 0$ ,  $I$  benefits from a competitive advantage in this Bertrand competition for the market and therefore corners the market in equilibrium. Moreover, ignoring weakly dominated strategies for  $E$ , the equilibrium is unique and such that  $F_E = f$  and  $F_I = f + s - \tau(m)$ , giving  $I$  a positive profit,  $\pi_I = s - \tau(m)$ , which moreover increases with  $m$ .

Suppose now that customer activism, say, is instead favourable to the entrant, i.e., consumers stick to  $E$  in case of multiple consumer responses. Then  $I$  wins the market only when  $F_I - F_E \leq s + \tau(m)$ ; therefore:

- When the switching cost is large enough, namely

$$s \geq -\tau(m),$$

---

<sup>20</sup>As usual with network effects, different expectations yield multiple consumer responses, which in turn may sustain multiple equilibria. The network effect arises here from on-net pricing rather than traditional club effects. In a different context, Matutes and Vives (1996) show that different expectations about the success of banks and coordination problems among depositors can result in multiple shared- and cornered-market equilibria (and even in a no-banking equilibrium).

then  $I$  still enjoys a competitive advantage and corners again the market in equilibrium; ignoring weakly dominated strategies, in equilibrium  $E$  sells at cost ( $F_E = f$ ) and  $I$  obtains a profit,  $\pi_I = s + \tau(m) (< s)$ , which decreases with  $m$ .

- When instead the switching cost is low ( $s < -\tau(m)$ ), the tariff-mediated network externalities dominate and customer activism gives a competitive advantage to  $E$ ; as a result, in all equilibria  $E$  corners the market.<sup>21</sup>

*Recap.* The above analysis can be summarized as follows. When  $m = 0$ , conditions (6) and (10) hold; therefore, from the above analysis,  $E$  cannot corner the market (this would require  $s < -3t$ , a contradiction), whereas  $I$  can corner the market only if the switching cost is prohibitively high, namely:

$$s \geq 3t.$$

When the switching cost is not that high,  $I$  may still corner the market when the termination charge departs from cost; however,  $E$  may then also corner the market. More precisely:

**Proposition 3** *Cornered-market equilibria exist in the following circumstances:*

- *Unique consumer response ( $\tau(m) > 0$ ):*
  - *Concave profits ( $\varphi(m) > 0$ ): there exists an equilibrium in which  $I$  corners the market when  $\psi(m) \leq s/3$ ; there also exists an equilibrium in which  $E$  corners the market when  $\psi(m) \leq -s/3$ .*
  - *Convex profits ( $\varphi(m) \leq 0$ ): there always exists an equilibrium in which  $I$  corners the market; there also exists an equilibrium in which  $E$  corners the market when  $\tau(m) \geq s$ .*
- *Multiple consumer responses ( $\tau(m) \leq 0$ ):*

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<sup>21</sup>In the limit case where  $s = -\tau(m)$ , both  $I$  and  $E$  can corner the market in equilibrium, but earn zero profit anyway.

- *Customer inertia favourable to the incumbent: there exists a unique equilibrium, in which  $I$  corners the market and its profit furthermore (weakly) increases with  $m$ .*
- *Customer activism favourable to the entrant: there generically exists a unique equilibrium; in this equilibrium,  $I$  corners the market when  $\tau(m) > -s$  (and  $I$ 's profit decreases with  $m$ ), whereas  $E$  corners the market when  $\tau(m) < -s$ .*

Building on this proposition, we have:

- For positive termination markups ( $m > 0$ ),  $\psi(m) > \varphi(m)$ , and both  $\tau(m)$  and  $\psi(m)$  decrease with  $m$ , as long as  $q(c+m) > 0$ . Therefore,  $I$  can corner the market when the access markup is so large that either  $\psi(m) \leq s/3$  (in which case (14) fails), or  $\tau(m) \leq 0$  (in which case (6) fails). In contrast,  $E$  cannot corner the market when  $I$  benefits from customer inertia in case of multiple consumer responses; and even if  $E$  benefits instead from customer activism, it cannot corner the market as long as  $\tau(m) > -s$ .
- For termination subsidies ( $m < 0$ ),  $\varphi(m) > \psi(m)$  and (6) holds, implying that there exists a unique, stable consumer response to prices. When  $\varphi(m) > 0$ , profits are concave and  $I$  can again corner the market when  $\psi(m) \leq s/3$ ; both  $E$  and  $I$  can corner the market, however, when  $\psi(m) \leq -s/3$ . When instead  $\varphi(m) < 0$ , profits are convex and  $I$  can always corner the market, whereas  $E$  can corner the market, too, only when  $\tau(m) \geq s$ .

## 4 Strategic choice of the access charge

When the switching cost is very high, namely  $s \geq 3t$ , the entrant cannot obtain any positive market share even under a cost-based access charge ( $m = 0$ ). The incumbent does however benefit from an increase in the access charge, as this further weakens the competitive pressure from its rival and generates greater profits: starting from  $m = 0$ , for which the stability condition (6) and the second-order condition (10) hold strictly, a slight



increase in the termination charge does not violate these conditions and still induces a cornered-market equilibrium; in this equilibrium,  $I$ 's profit is equal to

$$\pi_I^C(m) \equiv s - \tau(m) - mq(c + m),$$

which increases with  $m$  as long as the demand decreases:

$$\frac{d\pi_I^C}{dm} = -mq'(c + m) > 0.$$

When instead  $s < 3t$ , as we will assume in the rest of this section, the entrant successfully enters the market if the access charge is close to the termination cost ( $m \simeq 0$ ). However, departing significantly from cost-based access may allow the incumbent to corner the market. We now study in more detail this strategic incentive to alter the access charge in order to deter entry and increase the incumbent's profit.

## Foreclosure through high termination charges

Our extension of Gans and King's insight shows that increasing the termination charge degrades both operators' profits as long as the market remains shared. But further increasing the termination charge keeps the entrant entirely out of the market whenever tariff-mediated network externalities are sufficiently important, namely, whenever

$$\Delta \equiv v(c) - v(\infty) > t - \frac{s}{3}. \quad (29)$$

Indeed, under this condition, there exists a unique  $\check{m} > 0$  such that  $\psi(\check{m}) = s/3$ , and  $\psi(m) < s/3$  for any  $m > \check{m}$ . As long as  $\tau(m)$  remains positive (which may or may not be possible, since  $\tau(\check{m})$  can be positive or negative), increasing  $m$  above  $\check{m}$  then generates a unique equilibrium in which  $I$  corners the market and obtains again  $\pi_I^C(m)$ , which increases with  $m$ . However, if

$$\Delta > t, \quad (30)$$

then there exists a unique  $\hat{m} > 0$  such that  $\tau(\hat{m}) = 0$  and  $\tau(m) < 0$  for any  $m > \hat{m}$ . Raising  $m$  above  $\hat{m}$  then ensures that consumers always prefer to be all on the same network, but the profitability of this foreclosure strategy depends critically on which network is then more likely to win the market. For the sake of exposition, we will focus on two polar cases where, in case of multiple consumer responses, either customer inertia systematically favours the incumbent, or customer activism systematically favours the entrant.

**Customer inertia.** When  $I$  benefits from customer inertia, it can keep the entrant out and better exploit its market power by raising further the termination charge above  $\hat{m}$ ;  $I$  still wins the market and can charge up to (the subscript  $CI$  standing for "customer inertia")

$$F_I^{CI}(m) = f + s - \tau(m),$$

which increases with  $m$  as long as demand remains positive:

$$\frac{dF_I^{CI}}{dm} = q(c + m) \geq 0.$$

Therefore, the incumbent has an incentive to set  $m$  as high as possible, in order to extract consumer surplus without fearing any competitive pressure from the entrant. The only limitations come from consumer demand:

- Consumers may stop calling; raising  $m$  above  $\bar{m}$ , defined as the lowest value for which  $q(c + \bar{m}) = 0$ , does not increase  $I$ 's profit any further: for  $m > \bar{m}$ ,  $dF_I^{CI}/dm = 0$ .
- Consumers may also stop participating; there is no point insisting on larger termination markups than needed to sustain the monopoly level. If for example consumers' surplus  $v(c)$  is sufficiently "large" that even a pure monopoly prefers to maintain full participation, the optimal subscription fee extracts the full value from the farthest consumer ( $F^M = v(c) - t$ ); if  $\Delta$  is large enough,  $F^M$  can be sustained by setting the termination markup at  $m^M$  characterized by  $F_I^{CI}(m^M) = F^M$ , that is,

such that:

$$f + s - \tau(m^M) = v(c) - t,$$

or:<sup>22</sup>

$$v(c + m^M) = f + s.$$

Customer inertia, which could be interpreted as a form of "virtual" switching costs, is a good substitute for "real" switching costs. Indeed, in the presence of customer inertia, the incumbent can corner the market and earn the monopoly profit even in the absence of any real switching costs. As we will show below, however, in the presence of customer activism real switching costs are needed and determine equilibrium profits.

**Customer activism.** If instead customer activism favours the entrant in case of multiple consumer responses, then  $I$  never benefits from increasing the termination charge beyond  $\hat{m}$ , since the resulting profit then decreases with  $m$ . Its profit does increase in the range where  $I$  corners the market while (6) still holds (that is, for  $\check{m} \leq m \leq \hat{m}$ , where  $\check{m}$  is the positive solution to  $\psi(m) = s/3$ ), but, as noted above, it decreases in the range in which the market is shared (that is, for  $0 \leq m < \check{m}$ ). If  $\Delta$  is large enough (namely,  $\Delta > t$ ), to determine whether foreclosing the market is profitable for  $I$ , one should thus compare the profits obtained for  $m = \hat{m}$ , which is equal to

$$\hat{\pi}_I = s,$$

with the profit that could be obtained by sharing the market. In particular,  $\hat{\pi}_I = s$  should exceed the profit obtained for  $m = 0$ , which is equal to

$$\pi_I^0 = \frac{t}{2} \left(1 + \frac{s}{3t}\right)^2.$$

The comparison suffices to show that this foreclosure strategy cannot be profitable when

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<sup>22</sup>Indeed,  $m^M > \hat{m}$ : to ensure that even  $E$  would maintain full participation if it enjoyed a monopoly position,  $v(c)$  must exceed  $f + s + 2t$ , which implies

$$\tau(m^M) = t - v(c) + v(c + m^M) = t + f + s - v(c) < 0.$$

the switching cost is small, namely, when

$$s < \bar{s} \equiv (2 - \sqrt{3}) 3t, \quad (31)$$

that is, when even in the absence of any termination markup, the incumbent would keep less than about two-thirds of the market:

$$\alpha_I^0 = \frac{1}{2} \left( 1 + \frac{s}{3t} \right) < \frac{3 - \sqrt{3}}{2} \simeq 63\%.$$

## Foreclosure through large termination subsidies

Alternatively,  $I$  can try to foreclose the market by adopting a large subsidy ( $m \ll 0$ ). For  $m < 0$ , the stability condition (6) always holds, implying that there is a unique, stable, consumer response to prices (the issue of customer inertia or favoritism thus becomes irrelevant). Moreover,  $\varphi(m) = \psi(m) - mq(c+m)/6 \geq \psi(m)$ , which implies that profits are concave ( $\varphi(m) > 0$ ) whenever the shared market condition ( $\psi(m) > s/3$ ) is satisfied.

For a sufficiently large subsidy, one may have  $\psi(m) \leq s/3$ . However, as long as profits remain concave,  $I$ 's profit coincides again with  $\pi_I^C(m)$  and thus decreases when the size of the termination subsidy increases (in addition,  $E$  may as well corner the market if  $\psi(m) \leq -s/3$ ). Yet,  $I$  may benefit from increasing further the size of the subsidy, so as to make profits convex (i.e.,  $\varphi(m) \leq 0$ ); there is an equilibrium in which  $I$  corners the market and can charge up to  $F_I^{Conv} = f + \tau(m) + s$ , which increases with the size of the subsidy:

$$\frac{dF_I^{Conv}}{dm} = -q(c+m) < 0.$$

Foreclosing the market therefore requires subsidies that are large enough to make profits convex (i.e., to ensure  $\varphi(m) \leq 0$ ), which may be difficult to achieve:

- First,  $\varphi$  may remain positive: starting from  $m = 0$ , introducing a small subsidy *increases*  $\varphi$ , since  $\varphi'(0) = -q(c)/2 < 0$ ; while  $\varphi'(m) = (mq'(c+m) - q(c+m))/2$  may become positive for larger subsidies, there is no guarantee that this happens, and even in that case, there is no guarantee that  $\varphi$  may become negative for large

enough subsidies.

- Second, the size of subsidies may be limited by feasibility considerations; even "bill and keep" – i.e.,  $m = -c_T$  – may not suffice to generate a large enough subsidy.
- Third, very large subsidies and convex profits may allow the entrant, too, to corner the market; to avoid this, the incumbent should choose a termination charge satisfying  $\tau(m) < s$ , which, since  $\tau'(m) < 0$  for  $m < 0$ , imposes an additional restriction on the size of the subsidy (in particular, this restriction may be incompatible with  $\varphi(m) \leq 0$ ).
- Finally, subsidizing termination may generate abuses and, moreover, offering lower prices for off-net calls may not fit well with marketing strategies.

Despite these difficulties, large subsidies may in some cases allow the incumbent to corner the market and increase its profit. For example, if  $\varphi(m) < 0$  for the termination subsidy such that  $\tau(m) = s$ , then adopting this subsidy (or a slightly lower one) ensures that  $I$  corners the market and obtains a profit equal to  $s + \tau(m) = 2s$ , which is twice the maximal profit that  $I$  can obtain by foreclosing the market through a positive termination markup when customer activism benefits the entrant.

## Recap

The following proposition summarizes the above discussion:

**Proposition 4** *Suppose that  $s < 3t$ , so that cost-oriented access pricing would allow the entrant to share the market. While both networks would favour a small reduction in the access charge over a small increase in the access charge, the incumbent might increase its profit by departing further away from cost-based access pricing in order to corner the market; assuming that network externalities are large enough:*

- *If the incumbent benefits from customer inertia in case of multiple consumer responses, then it would have an incentive to increase the access charge as much as possible and could earn in this way up to the monopoly profit.*

- *If instead the entrant benefits from customer activism, then by foreclosing the market through a positive termination markup, the incumbent can earn a profit at most equal to  $s$ , which it can achieve by adopting  $m = \hat{m}$ , such that  $\tau(m) = 0$ ; the incumbent may also benefit from foreclosing the market through a large enough termination subsidy, although feasibility, strategic (equilibrium multiplicity) and marketing considerations tend to limit this possibility.*

**Illustration: linear demand function.** Suppose that the utility function takes the form

$$u(q) = aq - \frac{b}{2}q^2,$$

with  $a, b > 0$ . The demand function is then linear,  $q(p) = (a - p)/b$ , while consumer's surplus is  $v(p) = (a - p)^2/2b$ . We adopt the parameter values of De Bijl and Peitz (2002, 2004):  $a = 20$  euro-cents,  $b = 0.015$  euro-cent,  $c_T = 0.5$  euro-cent,  $c = c_O + c_T = 2$  euro-cents, and  $t = 35$  euros.<sup>23</sup> The feasible range for the termination markup is thus  $m \geq -c_T = -0.5$  euro-cent and, in this range, it can be checked that  $\varphi$  and  $\psi$ , as well as  $\tau$ , are all decreasing in  $m$ . In particular, condition (6) is satisfied for  $m < \hat{m} = 3.2014$  euro-cent, in which case the second-order condition  $\varphi(m) > 0$  is also satisfied. In addition, the shared-market condition (14),  $\psi(m) > s/3$ , amounts to  $m < \check{m}(s)$ , where  $\check{m}(s)$  decreases with  $s$ . Therefore, for any  $s < 3t$  (so as to ensure that the market would be shared for  $m = 0$ , that is,  $\check{m}(s) > 0$ ), the market is always shared whenever access is subsidized ( $m < 0$ ) or moderately priced (that is,  $m < \min\{\hat{m}, \check{m}(s)\}$ ); the incumbent can however corner the market by insisting on a large enough access markup ( $m > \min\{\hat{m}, \check{m}(s)\}$ ).<sup>24</sup> It can moreover be checked that, in the limited admissible range of negative values for  $m$ , the incumbent's (shared-market) equilibrium profit decreases with  $m$ ; "bill and keep" (that is,  $m = -c_T = -0.5$  euro-cent) thus constitutes the most profitable access agreement in this range. Below we compare this profit with the profit that the incumbent can achieve by cornering the market through large access markups. To complete the welfare analysis

<sup>23</sup>In De Bijl and Peitz (2002),  $t = 60$  euros, whereas in De Bijl and Peitz (2004),  $t = 20$  euros. Since this parameter is difficult to measure, its value is based on experience obtained in the test runs of their model. Adopting  $t = 35$  euros ensures that  $v(c) > 3t$ .

<sup>24</sup>By contrast,  $E$  cannot corner the market in the absence of customer activism, since (6) here implies (14).

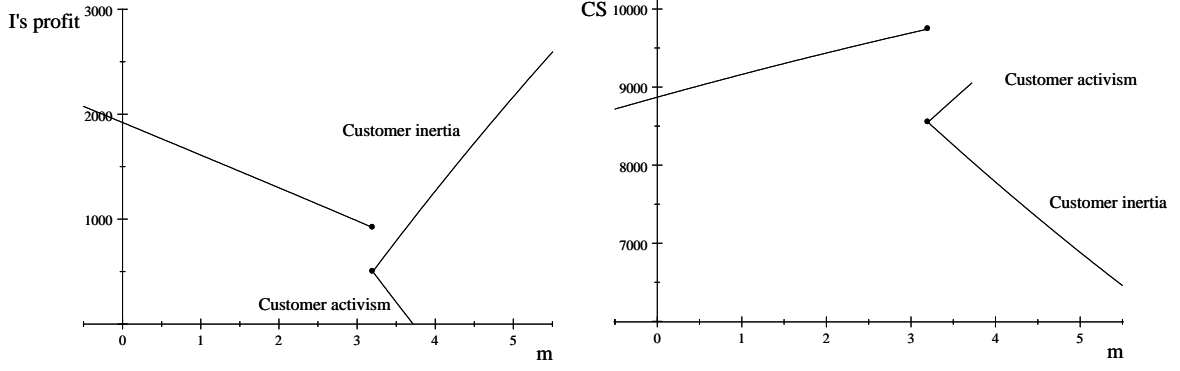


Figure 6: Incumbent's equilibrium profit for small switching costs:  $s = 5$  euros.

we also study the impact of the access charge on consumer surplus ( $CS$ ), net of fixed fees and switching and transport costs:

$$\begin{aligned}
 CS = & \alpha_I (\alpha_I v(c) + \alpha_E v(c + m) - F_I) + \alpha_E (\alpha_E v(c) + \alpha_I v(c + m) - F_E) \\
 & - \int_0^{\alpha_I} t x dx - \int_{\alpha_I}^1 t(1 - x) dx - s \alpha_E.
 \end{aligned}$$

For illustrative purposes, we consider two polar cases: *i*) small switching costs:  $s = 5$  euros; *ii*) large switching costs:  $s = 70$  euros.

- Small switching costs: We have  $\check{m}(s) = 6.98 > \hat{m} = 3.2$ . Therefore, for  $m < \hat{m}$  the market is shared between the two networks whereas for  $m \geq \hat{m}$ , there are multiple consumer responses. In that latter range,  $I$  corners the market; if it moreover benefits from customer inertia, its profit increases with  $m$  and, for  $m$  large enough, exceeds the profit achieved when sharing the market under lower access charges. In case of customer activism, however,  $I$ 's profit decreases with  $m$ , as illustrated in Figure 6 – and  $E$  moreover corners the market when  $m$  becomes large enough (namely, when  $m \geq 3.72$ , where  $\tau(m) \leq -s$ ). In addition,  $I$ 's profit from cornering the market through  $\hat{m}$ ,  $\pi_I = s$ , is lower than in any shared-market equilibrium. Thus,  $I$  would here choose to foreclose the market through large access markups only when it benefits from customer inertia.
- Large switching costs: We now have  $\check{m}(s) = 2.71 < \hat{m} = 3.2$ . Therefore, for  $m < \check{m}$

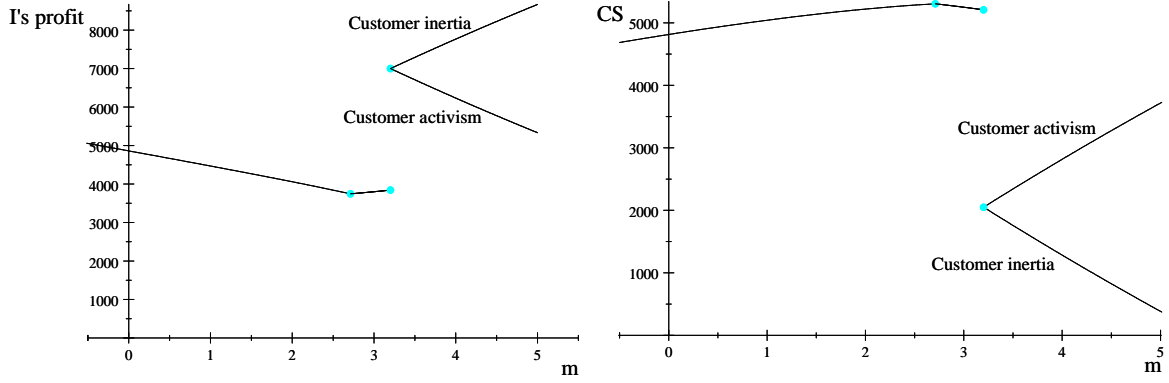


Figure 7: Incumbent's equilibrium profit for large switching costs:  $s = 70$  euros.

the two networks share the market, whereas for  $m$  between  $\tilde{m}$  and  $\hat{m}$   $I$  corners the market (even though there is a unique consumer response and profit functions are concave) by charging  $F_I = f + s - \tau(m) - mq(c + m)$ . In this equilibrium,  $I$ 's profit increases with  $m$ . For  $m > \hat{m}$ , there are multiple consumer responses and  $I$  still corners the market, although its profit increases with  $m$  only if it benefits from customer inertia, as illustrated by Figure 7.  $I$ 's profit from cornering the market with  $m = \hat{m}$  is now higher than in any shared-market equilibrium (even with "bill and keep"), however. Therefore, even in case of customer activism,  $I$  will here prefer to corner the market with a large enough access markup (namely,  $\hat{m}$ ) rather than sharing the market with lower or below-cost access charges.

*Consumer surplus.* In both cases (for small and large switching costs), consumer surplus increases with  $m$  as long as the networks share the market. The reason is that competition is more aggressive for higher access charges. Also, in both cases, the incumbent corners the market when  $m \geq \hat{m}$  and consumer surplus then decreases (increases) with  $m$  in the presence of customer inertia (activism), since a higher  $m$ , reduces (increases) the competitive pressure of the entrant. Finally, in the case of large switching costs, the incumbent also corners the market when  $m$  lies between  $\tilde{m}$  and  $\hat{m}$ , and in this range increasing the access charge reduces the competitive pressure, allows the incumbent to charge a higher fixed fee and thus results in lower consumer surplus.



## 5 No termination-based price discrimination

In this section we examine whether the incumbent can foreclose competition through access charges when there is no termination-based price discrimination. Network  $i$ 's profit is then (for  $i \neq j = I, E$ ):

$$\pi_i = \alpha_i [(p_i - c)q(p_i) + F_i - f + \alpha_j m(q(p_j) - q(p_i))].$$

A detailed analysis of shared-market equilibria can be found in Carter and Wright (2003) and López (2007). Market shares are given by:

$$\alpha_I(w_I, w_E) = 1 - \alpha_E(w_I, w_E) = \frac{1}{2} + \sigma(w_I - w_E - s),$$

where  $w_i = v(p_i) - F_i$  denotes the net surplus that operator  $i$  offers its customers. We can interpret network  $i$ 's strategy as offering a price  $p_i$  and a net surplus  $w_i$  and, given network  $j$ 's strategy, network  $i$ 's best response moreover entails

$$p_i = \tilde{p}_i(w_i) = c + \tilde{\alpha}_j(w_i) m. \quad (32)$$

Therefore, given network  $j$ 's strategy, we can write network  $i$ 's profit as

$$\tilde{\pi}_i(w_i) = \tilde{\alpha}_i[v(\tilde{p}_i) - w_i - f + \tilde{\alpha}_j m q(p_j)],$$

with

$$\begin{aligned} \tilde{\pi}'_i(w_i) &= \sigma[v(\tilde{p}_i) - w_i - f + (\tilde{\alpha}_j - \tilde{\alpha}_i) m q(p_j) + \tilde{\alpha}_i m q(\tilde{p}_i)] - \tilde{\alpha}_i, \\ \tilde{\pi}''_i(w_i) &= -\sigma[2 + 2\sigma m(q(p_j) - q(\tilde{p}_i)) + \tilde{\alpha}_i \sigma m^2 q'(\tilde{p}_i)]. \end{aligned}$$

For  $m = 0$ ,  $\tilde{\pi}''_i(w_i) = -2\sigma < 0$  and second-order conditions therefore hold. First-order conditions yield  $p_I = p_E = c$  and

$$\alpha_I^s(0) = \frac{1}{2} \left(1 + \frac{s}{3t}\right),$$

so a shared-market equilibrium exists provided that  $s < 3t$ , in which case the incumbent's profit is equal to

$$\pi_I^s(0) = \frac{t}{2} + \frac{s}{3} \left(1 + \frac{s}{6t}\right).$$

We also know from the previous papers that any small departure from  $m = 0$  lowers the incumbent's profit.

Consider now a candidate equilibrium in which  $I$  corners the market. In the light of the above analysis, it follows that  $p_I = c$  and  $p_E = c + m$ . For this to be an equilibrium, even the consumers closest to  $E$  must prefer to stay with  $I$ , that is,  $v(c) - t - F_I \geq v(c + m) - s - F_E$ ; and since  $I$  maximizes its profit, this inequality cannot be strict, therefore:

$$F_I = F_E - \tau(m) + s. \quad (33)$$

Moreover,  $I$  should not gain from a marginal increase in its fee:

$$0 \leq \tilde{\pi}'_I(w_I)|_{\alpha_I=1} = \sigma [F_I - f + m(q(c) - q(c + m))] - 1,$$

that is:

$$F_I \geq f + 2t - m(q(c) - q(c + m)). \quad (34)$$

In addition,  $E$  should not make any profit by stealing a few consumers, that is:

$$F_E - f + mq(c) \leq 0. \quad (35)$$

Using (33), we can rewrite conditions (34) and (35) as:

$$f - mq(c) \geq F_E \geq f + 2t + \tau(m) - s - m(q(c) - q(c + m)). \quad (36)$$

Conversely, any  $F_E$  in the above range can support a cornered-market equilibrium if second-order conditions are moreover satisfied; eliminating weakly dominated strategies singles out the equilibrium in which  $F_E = f - mq(c)$ ,  $F_I = f - \tau(m) - mq(c) + s$  and

network  $I$ 's profit is equal to:

$$\pi_I^c(m) = s - \tau(m) - mq(c).$$

This expression is maximal for  $m = 0$ , where it is equal to  $\pi_I^c(0) = s - t$ . Therefore, when  $s > 3t$ , in which case there is no shared-market equilibria and thus  $I$  always corners the market,  $I$ 's profit is maximal for  $m = 0$  (and the above-described cornered-market equilibrium indeed exists, since second-order conditions are always satisfied for  $m = 0$ ). We now show that, when  $s < 3t$ ,  $I$  cannot gain from departing from  $m = 0$  in order to corner the market. It suffices to show

$$\pi_I^c(0) = s - t < \pi_I^s(0) = \frac{t}{2} + \frac{s}{3} \left(1 + \frac{s}{6t}\right),$$

which amounts to:

$$\lambda(s) \equiv \frac{s}{3} \left(2 - \frac{s}{6t}\right) - \frac{3t}{2} < 0.$$

Since  $\lambda(3t) = 0$  and  $\lambda'(s) > 0$  (since  $s < 3t$ ), it follows that  $\lambda(s) < 0$  for  $s < 3t$ .

Consider now a candidate equilibrium in which  $E$  corners the market, then  $p_I = c + m$  and  $p_E = c$ . Moreover, the pair of prices  $(F_I, F_E)$  must satisfy

$$v(c + m) - F_I \leq v(c) - F_E - s - t.$$

In addition,  $I$  should not make any profit by attracting a few customers, i.e.,

$$F_I \leq f - mq(c).$$

But combining those two conditions yields

$$\pi_E = F_E - f \leq v(c) - v(c + m) - mq(c) - s - t,$$

where the right-hand side is maximal for  $m = 0$ , where it is equal to  $-s - t < 0$ . Therefore,

in the absence of termination-based price discrimination the entrant cannot corner the market.

## 6 Competition under the Receiver Pays regime and no termination-based price discrimination

In many European countries networks do not charge for receiving calls even when it is not explicitly forbidden by NRAs. In contrast, in the United States mobile network operators usually charge their subscribers for the calls they receive. The reason may be an endogenous price response to the level of the termination charge, i.e., low termination charges in the U.S. may induce networks to charge their customers for receiving calls so as to recover their cost. Cambini and Valletti (2007) show for example that when there exist interdependencies between incoming and outgoing calls, operators charge for reception only when termination charges are low enough.

This section builds on Jeon, Laffont and Tirole (2004) and López (2008), where subscribers derive a surplus from making and receiving calls, and networks offer a three-part tariff:  $\{F_i, p_i, r_i\}$ , where  $r_i$  denotes the per-unit reception charge. Thus, termination-based price discrimination is not allowed.<sup>25</sup> Let  $\mu(q)$  denote the utility from making  $q$  calls, and  $\tilde{\mu}(\tilde{q})$  denote the utility from receiving  $\tilde{q}$  calls. For a given  $p_i$  the caller's demand is given by  $\mu'(q) = p$ , whereas for a given  $r$  the receiver's demand is given by  $\tilde{\mu}'(\tilde{q}) = r$ ;<sup>26</sup> assuming that receivers are allowed to hang up, the volume of calls from network  $i$  to network  $j$  is then  $Q(p_i, r_j) = \min\{q(p_i), \tilde{q}(r_j)\}$ . In order to make the analysis tractable, those papers assume that i) the caller's and receiver's utilities are subject to a random noise, which smoothes the demand,<sup>27</sup> and ii) the caller's and receiver's utilities are additively separable with respect to the random noise:  $u = \mu(q) + \varepsilon q$  and  $\tilde{u} = \tilde{\mu}(\tilde{q}) + \tilde{\varepsilon} \tilde{q}$ , where

<sup>25</sup> Jeon, Laffont and Tirole (2004) show that allowing networks to charge different calling and reception charges according to whether the call is on- or off-net, creates strong incentives for connectivity breakdown through infinite calling or reception charges (even among equal networks).

<sup>26</sup> As usual, these utility functions are twice continuously differentiable, with  $\mu' > 0$ ,  $\mu'' < 0$ ,  $\tilde{\mu}' > 0$ , and  $\tilde{\mu}'' < 0$ .

<sup>27</sup> More specifically, in Jeon, Laffont and Tirole (2004) only the receiver's utility is subject to a random noise, which is enough to smooth the demand. López (2008) generalizes their setup by allowing a random noise in both the callers' and receivers' utilities. We are considering this more general setup.

$\varepsilon$  and  $\tilde{\varepsilon}$  denote, respectively, the random shocks on the caller's and receiver's utilities.

Consumers learn the realization of  $\varepsilon$  and  $\tilde{\varepsilon}$  only after their subscription decisions, which they thus base on expected volumes. Both papers show that charging calls and receptions at the off-net cost is a candidate equilibrium:

$$p_i = c + m, \quad r_i = -m.$$

López (2008) extends the analysis to asymmetric installed bases and positive switching costs, and moreover shows that this off-net-cost pricing equilibrium exists and is the unique possible equilibrium when networks are relatively poor substitutes and the random noise has a wide enough support. In addition, when setting usage prices at the off-net cost, network  $i$ 's profit writes as:

$$\hat{\pi}_i = \alpha_i(F_i, F_j)[F_i - f],$$

where  $\alpha_i(F_i, F_j) = 1/2 + (2\alpha_i^0 - 1)\sigma s - \sigma(F_i - F_j)$ . Since  $\hat{\pi}_i$  does not depend on  $m$ , it follows that the access markup has no impact on networks' equilibrium fixed fees, and thus on networks' profits. This profit-neutrality result implies that, in the absence of termination-based price discrimination, networks cannot use access charges to soften or foreclose competition when they compete in three-part tariffs. The reason is that for any given access markup and installed bases of customers, the operators always find it optimal to set usage prices at the off-net cost, which in turn neutralizes the impact of the access charge on profit.

## 7 Conclusion

We have studied the impact of reciprocal access charges on entry when consumers face switching costs, and networks compete in three-part tariffs, charging possibly different prices for off-net calls. The analysis supports the conventional wisdom that established networks prefer high access charges. In particular, when the incumbent benefits from

customer inertia, it has an incentive to insist on the highest possible (reciprocal) access markup, so as to foreclose the market and exploit fully the resulting monopoly power; a large termination subsidy could also achieve the same outcome, although subsidies may in practice be limited by feasibility constraints and moreover trigger various types of arbitrage. This possibility of successful foreclosure supports a call for regulatory authorities to set bounds on access markups (and subsidies).

The scope for foreclosure is more limited if the entrant benefits instead from customer activism; while the incumbent can still wish to manipulate the termination charge in order to prevent entry, too high access charges might then allow the entrant to overtake the incumbent. As a result, optimal foreclosure strategies rely either on limited access markups or on access subsidies, and are profitable only when consumers' switching costs are large enough.

Irrespective of whether customers tend to favour the incumbent or the entrant in case of multiple potential responses to networks' prices, foreclosure strategies are profitable here only when they result in complete entry deterrence: while the incumbent can increase its market share by insisting on above-cost reciprocal charges, this also results in more intense price competition and, as a result, both operators' equilibrium profits are lower than when the reciprocal access charges are at or below cost. In other words, limiting entry without deterring it entirely is never profitable.

Finally, the network effects created by termination-based price discrimination appear to be a key ingredient for profitable foreclosure strategies. Indeed, in the absence of on-net pricing, neither the incumbent nor the entrant find it profitable to manipulate the access charge so as to foreclose competition. In addition, in a receiver pays regime, neither operator can use the access charge to foreclose competition.

## 8 APPENDIX

**Proof of Proposition 2.** Using (8) and (11), network  $i$ 's profit can be written as

$$\pi_i = \frac{\varphi(m)}{2} (2\alpha_i)^2,$$

where  $\varphi(m) > 0$  (from (10)). Replacing (13) into this expression yields

$$\pi_i(m) = \frac{\varphi(m)}{2} \left( 1 + \frac{\delta_i s}{3\psi(m)} \right)^2. \quad (37)$$

For the sake of exposition, we will assume that  $q(c+m)$  remains positive; it is easy to extend to the case  $q(c+m) \geq 0$ .<sup>28</sup>

It is straightforward to check that, for  $m > 0$ , both  $\varphi$  and  $\psi$  decrease with  $m$ . It follows that  $E$ 's profit decreases with  $m$  when  $m > 0$  (since both  $\varphi$  and  $2\alpha_E = 1 - s/3\psi(m)$  decrease with  $m$ ).

We now show that  $I$ 's profit satisfies  $\pi_I(m) < \pi_I(0)$  for any  $m > 0$ . Since  $\delta_I = 1$  and  $\psi(m) = \varphi(m) + mq(c+m)/6 > \varphi(m)$ , we have:

$$\pi_I(m) = \frac{\varphi(m)}{2} \left( 1 + \frac{s}{3\psi(m)} \right)^2 < \Psi(m) \equiv \frac{\psi(m)}{2} \left( 1 + \frac{s}{3\psi(m)} \right)^2,$$

where

$$\Psi' = \frac{d}{d\psi} \left[ \frac{\psi}{2} \left( 1 + \frac{s}{3\psi} \right)^2 \right] \psi' = \frac{1}{2} \left( 1 + \frac{s}{3\psi} \right) \left( 1 - \frac{s}{3\psi} \right) \psi' = 2\alpha_I (1 - \alpha_I) \psi' < 0,$$

since  $\alpha_I \in (0, 1)$  and  $\psi'(m) = -[q(c+m) - 2mq'(c+m)]/3 < 0$ . Therefore,

$$\Psi(m) < \Psi(0) = \pi_I(0).$$

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<sup>28</sup>For  $m$  large enough,  $q(c+m)$  may become zero;  $\tau$ ,  $\psi$ ,  $\varphi$ ,  $\alpha_i$  and  $\pi_i$  then remain constant as  $m$  further increases and the analysis below still applies to the range of  $m$  over which  $q(c+m) > 0$ .

Similarly, for  $m < 0$  we have  $\psi(m) < \varphi(m)$  and thus:

$$\pi_I(m) > \Psi(m).$$

Since  $\Psi(0) = \pi_I(0)$  and  $\Psi'(0) = -2\alpha_I(0)(1 - \alpha_I(0))q(c)/3 < 0$ ,  $\pi_I(m) > \pi_I(0)$  for  $m$  slightly negative. ■



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