Resale Price Maintenance under Asymmetric Information*

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Abstract

We study Resale Price Maintenance (RPM) and quantity fixing arrangements (QF) in a framework with successive monopolies under both adverse selection and moral hazard. The analysis compares the private and the welfare properties of both contractual modes. Under asymmetric information, both kinds of vertical contracts entail a *double marginalization* driven by the information rents distributed to a privately informed downstream retailer. This forces the upstream producer to sell above his marginal costs. The upstream producer always prefers RPM to QF, but the impact of RPM on consumers' surplus is ambiguous. Whenever RPM is the preferred contracting mode for the vertical structure from an ex ante viewpoint, it also raises consumers' surplus, thereby producing a Pareto improvement relative to QF contracts.

Keywords: double marginalization, incomplete information, resale price maintenance, quantity forcing, vertical control, vertical restraints.

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1 Introduction

Contracts between vertically related firms (manufacturer-retailer pairs) have been recently at the heart of an intense debate in the IO literature. A number of theoretical contributions[1] have demonstrated that an appropriate design of vertical restraints allows upstream manufacturers to influence the competitive behavior of their retailers. A central issue for competition policy is then to study the welfare properties of those vertical arrangements which are usually observed in real-world environments[2] and which, potentially, may restrain the retailers' activities. In this respect, two important features of a vertical relationship (manufacturer-retailer) might play a crucial role in order to clarify what are the welfare properties of different kinds of vertical restraints. First, retailers are typically better informed about local market conditions than manufacturers, so that an adverse selection problem arises. Second, retailers carry out demand enhancing (non-market) activities such as marketing and advertising policies which are typically not verifiable by third parties; a moral hazard issue.

This paper examines both the private and the welfare properties of vertical contracts based on retail price restrictions in a framework with successive monopolies. We consider a setting where information asymmetries constrain the efficiency frontier of a contractual relationship between an upstream manufacturer and a privately informed downstream retailer. These risk-neutral monopolists interact in a standard principal-agent way under both adverse selection and moral hazard. The upstream firm (principal) has full bargaining power and hires a retailer (agent) before production occurs, but after uncertainty about demand is realized. Two types of vertical contracts are compared. The upstream manufacturer can either commit to a simple *quantity fixing* (QF) contract, or to a more sophisticated arrangement, comparable to *resale price maintenance* (RPM). A QF arrangement is more flexible than RPM in the sense that, beyond fixing the quantity supplied to final consumers, it leaves the downstream firm *free* to choose its most preferred level of demand enhancing marketing activities (promotional expenditures and/or production of indivisible services). Instead, a RPM mechanism, besides fixing the quantity supplied in the final market, also restrains the retail price charged to final consumers in order to *indirectly control* the non-market activities exerted by the retailer.

Asymmetric information plays a crucial role in our analysis. It introduces two sources of inefficiency. First, because of the retailer's superior information, the upstream firm has to give up information rent to induce information revelation: a *distributive effect.* Second, when both the input and the retail price influence these information rents, an *allocative*

[1] Blair and Lewis (1994), Gal-Or (1991), Martimort (1996), Mathewson and Winter (1984), Jullien and Rey and (2000), Rey and Tirole (1986) and Winter (1993) among others.

[2] Exclusive territories, resale price maintenance, exclusive dealings etc. are forms of vertical contracts typically banned by antitrust law.

effect reduces *productive efficiency.*[3] In this case, to improve productive efficiency and minimize agency costs, optimality requires a downward distortion of the inefficient types' allocations. Importantly, this effect generates a peculiar form of *double marginalization* (or excessive retail pricing) which is at the heart of the net welfare loss that we shall analyze in the sequel.

The objective of the analysis is twofold. We first study the private and welfare properties of both types of vertical restraints. The use of retail price restrictions may have an *ambiguous* effect on consumers' welfare, though it raises unambiguously manufacturer's profits relative to QF. Precisely, under asymmetric information, both types of vertical restraints force the upstream manufacturer to sell above his marginal costs, thereby producing *double marginalization.* This effect not only distorts (downward) productive efficiency but also lowers consumers' welfare. In this respect, we provide conditions under which the welfare loss that double mark-ups inflict on consumers is minimized either by RPM or QF. Our results show that RPM enhances consumers' surplus relative to QF whenever the retailer's marginal utility of effort is convex. In light of these results, the analysis reveals that the concerns about social desirability of RPM, typically addressed by antitrust authorities, are *justified* only under certain technological conditions related to the nature of services produced by retailers.

Second, we investigate conditions under which an antitrust authority should ban contracts based on price restrictions. We show that, whenever a contracting mode maximizes consumers' surplus it also maximizes the joint-profit of the vertical structure. Assessing the social desirability of a given vertical restraint amounts thus only to verify whether it is also privately optimal from the vertical structure's point of view.

The present paper contributes to a large theoretical and empirical literature on vertical restraints, dating back to the seminal contributions by Spengler (1950) and Telser (1960). This literature focuses on the private and welfare effects of vertical restraints and is motivated by the extensive use of those practices in the real-world. Remarkably, the use of retail price restrictions has been subject to radical changes in intellectual and political attitudes over the last decades. The economic effects of these practices have been at the heart of a controversial debate for a long time. As stressed by Neven et al. (1998), in the early days of competition policy free-market transactions were treated as a necessary condition for a market to achieve social efficiency. According to this simple view, any restriction to the business strategies of downstream firms was perceived as socially inefficient and "presumably" anticompetitive. By contrast, the so-called *Chicago School approach,* stemming from the pioneering contributions by Spengler (1950) and Telser (1960) and

[3] In our setting we shall say that contract A entails more productive efficiency relative to contract B if the quantity supplied on the final market under A is closer to a given benchmark relative to the quantity supplied under B.

developed by the work of Mathewson and Winter (1984-1986), subsequently proposed an argument in favor of vertical restraints. Intuitively, this body of literature pointed out that vertical restraints in the form of retail price targets may instead beneficially help upstream manufacturers to internalize transaction costs.[4] The traditional (Chicago style) argument in favor of RPM hinges on the simple idea that, when only linear prices are allowed, it prevents the social loss due to *double marginalization*[5] and, likewise, it allows vertical organizations to improve productive efficiency. More recently, a number of contributions, noticeably Tirole and Rey (1986) and Jullien and Rey (2000) among others, have noted that, under some significant circumstances, the Chicago conjecture may fail to the extent that: (i) under uncertainty and/or informational asymmetries price restrictions might generate monopoly power; and (ii) in several important cases they harm the competitive process by softening competition either at the manufacturer or at the retail level.[6] Our main contribution consists in showing that those results may follow from a simple successive monopolies model under asymmetric information.[7]

The substantial discrepancy between the Chicago school conjecture and the insights of the more recent theoretical contributions has certainly disoriented competition policy authorities. For instance, the U.S. Supreme Court has changed several times its view about the lawfulness of RPM.[8] The main source of this inconsistency seems to be a case-by-case analysis of circumstances where vertical restraints are adopted. This is the reason why many economists, as for instance Tirole (1988), recognize that a crucial step in this matter would be the provision, at a theoretical level, of a careful classification and operational criteria clarifying under what conditions certain vertical restraints are likely to harm third-parties such as consumers. In this perspective, the second part of our analysis strengthens the conjecture that vertical contracts based on retail price restrictions should not be *banned,* provided that contractual modes maximizes the ex ante joint-profit of the vertical structure. This suggests that, in some important circumstances, antitrust

[4]Such as vertical and horizontal externalities harmful to final consumers and downstream competitors.

[5]Under the hypothesis of complete information, double mark-ups exist only if firms are forced to employ linear pricing. In this setting the upstream firm will charge a wholesale price above its marginal cost which, in turn, implies excessive retail pricing.

[6]In particular, Jullien and Rey (2000) emphasize that the use of RPM enhances market power because eliminating retail price flexibility facilitates collusive behavior in imperfect competitive industries. In the same vein, Whinston (1990) shows that certain forms of vertical restraints may harm consumers and produce anticompetitive effects in the presence of dynamic externalities.

[7]Kastl and Piccolo (2004) extends a version of this model without moral hazard to a imperfect competitive environment where two vertical hierarchies produce differentiated goods.

[8]Already in the 1911 the Supreme Court declared this practice to be a per se violation of the Sherman Act as a form of horizontal price fixing among downstream competitors (Dr. Miles Medical Case). This position changed just eight years later (1919) when the same Court softened its previous conclusions. In 1968 the court still pronounced against RPM when a newspaper attempted to impose maximum resale price on its distributors. Finally in 1997, the court revised its previous positions by declaring that maximum RPM should be judged under the rule of reason even if minimum RPM was still declared illegal per se. See Comanor (1985) for an extensive discussion on this issue.

law should treat retail price maintenance as a welfare-improving instrument. This is in contrast with the past antitrust authorities' tendency to sentence retail price restrictions and protect simple arrangements such as quantity discounts contracts.[9]

Related Literature. Before proceeding, let us briefly relate our analysis to the literature on vertical contracting. Other previous contributions have analyzed the welfare effects of retail price restrictions in a way close to ours. Gal-Or (1991) compares vertical contracts based on quantity fixing and price restrictions in a context of successive monopolies where adverse selection can either be unidimensional or two-dimensional according to the type of instruments used by the upstream manufacturer in order to extract the downstream retailer information. In this pure adverse selection context, she points out that the use of price restrictions reduces the "dimensionality" of the adverse selection problem and, in contrast to our results, besides mimicking vertical integration, it *always* improves upon productive efficiency. This result implies, in turn, that RPM increases *unambiguously* consumers welfare through alleviating double marginalization. Introducing a moral hazard component, we will show that this result must be qualified and depends on fine details of the retailers' technology for providing services.

Blair and Lewis (1994) consider also a vertical contracting problem with both adverse selection and moral hazard for a general demand specification. Differently from us, however, they focus on RPM contracts, and show that optimal contracts exhibit some form of resale price maintenance and quantity fixing. They do not compare those contracts with QF arrangements and thus cannot draw any implications for antitrust policy as we do below.

Our contribution extends the results of Gal-Or and Blair-Lewis in two respects. On the one hand, as Gal-Or, we compare the welfare and private effects of different vertical restraints, namely QF and RPM contracts, but we introduce the moral hazard component driving our welfare results. As we will show, adding this moral hazard component is key to our result. Not only it is certainly more relevant from a real world perspective, but it also allows us to explore more carefully the relationship between the kinds of contractual instruments used by upstream producers, retailer's information rents and double marginalization.[10] On the other hand, while Blair and Lewis are mainly interested in comparing RPM to a contract maximizing joint-profits, we are more concerned with the differences between RPM and QF. In doing so we shed light on two important aspects of the problem at hand. First, on a positive ground, we provide a careful, formal charac-

[9] Over the last decades the antitrust authorities in the Unites States has argued unambiguously against the use of resale price maintenance. Refiners, for instance, have been a favorite target of antitrust arrangements. Courts decisions have pronounced as unlawful contractual schemes through which the retail price was controlled by the upstream refiners.

[10] Without moral hazard, our model, as in Gal-or (1991), would yield the extreme result that RPM suffices to nullify the scope for retailers to get information rents from their private information on demand.

terization of the private and welfare properties of both contractual regimes. Second, our analysis also provides some criteria that could be used by antitrust authorities.

The paper is organized as follows. Section 2 sets up the model. Section 3 briefly studies the complete information benchmark and confirms the well-known equivalence between contracting modes in such contexts. Section 4 characterizes the optimal allocations under both kinds of mechanisms in a context of asymmetric information and compares the main features of those contracts. Section 5 analyzes the welfare properties of both contracting modes. Section 6 provides some extensions. Section 7 briefly summarizes our findings. Most of the proofs are relegated to an Appendix.

2 The Model

Setting. Consider an upstream manufacturer who sells an essential input to a privately informed retailer. Assume that this downstream firm, a monopolist as well, converts this intermediate input with a one-to-one technology[11] and supplies the final product on a retail market. Let $P(q,\tilde{\theta},e) = \tilde{\theta} + e - q$ denote the inverse market demand.[12] This simple specification captures two traditional features of manufacturer-retailer relationships. First, consumers must be informed about the existence and the characteristics of a product before buying it; hence e denotes a nonverifiable demand-increasing effort[13] exerted by the retailer. Alternatively, the retailer may exert after-sales services which are to a large extent non-contractible. Providing effort is costly, and we denote by $\psi(e)$ an increasing, strictly convex and three times differentiable disutility function. Second, retailers have more information about final demand than manufacturers. Accordingly, we model $\tilde{\theta}$ as an exogenously given random variable measuring the consumers' *willingness to pay*[14], distributed on the compact support $\Theta \equiv \left[\underline{\theta}, \overline{\theta}\right]$ with a cumulative distribution function $F(\theta)$ and a positive density $f(\theta)$. We will sometimes refer to θ as the retailer's type.

Assumptions. Let $h(\theta) = (1 - F(\theta))/f(\theta)$ define the inverse hazard rate associated to $F(\theta)$.

[11] This is a simplifying assumption used extensively in the IO literature.

[12] Up to a simple renormalization, our model can be generalized to allow inverse demand functions of the form $P(q,\tilde{\theta},e) = \theta + e - \phi(q)$ for some $\phi(\cdot)$ increasing and convex.

[13] This variable can be thought of as a nonverifiable non-market activity carried out by the downstream firm such as expenditures on advertising and/or production of demand-increasing indivisible services. By nonverifiable here we mean that the effort level is not verifiable by a third-party, such as a Court, enforcing contracts.

[14] This variable might represent local market conditions which are observed only by the closest firm to final consumers.

A1. (i) The cumulative distribution function $F(\theta)$ is differentiable; (ii) $f(\theta) > 0$ and $\dot{h}(\theta) = \partial h(\theta)/\partial\theta \leq 0$ for all θ with $\underline{\theta} \geq 1/f(\underline{\theta})$.

These assumptions are standard in the agency literature under adverse selection. In particular, (i) is not necessary but is assumed for convenience[15]; whereas (ii) ensures full separation of types at the optimum.

A2. (i) $\psi(0) = 0$ and $\psi'(e) \geq 0$ for all $e \geq 0$; (ii) Inada conditions: $\psi'(0) = 0$ and $\psi'(e) = +\infty$ as $e \to +\infty$; (iii) $\psi''(e) > 1/2$ for all $e \geq 0$.

These are technical requirements needed in order to deal with well-defined (concave) optimization programs having interior solutions.

A3. Both the upstream manufacturer and the downstream retailer produce at zero (marginal) costs.[16]

Contracts. As in the previous literature (Blair and Lewis (1994) and Gal-Or (1991)), we invoke the *Revelation Principle*[17] to describe the set of incentive feasible allocations. Precisely, the model considers the standard framework where a communication stage between the upstream manufacturer (principal) and the downstream firm (agent) is played before production occurs. At this stage the informed agent delivers to the uninformed principal a message, $\hat{\theta}$, about the realized state of demand. Given this message, the manufacturer proposes a contract specifying, for all $\hat{\theta}$, both a quantity, a franchise fee and, possibly, a retail price restriction in the form of a price *target*. Let $\mathcal{M} \equiv \{QF, RPM\}$ define the space of deterministic and piecewise differentiable[18] direct revelation mechanisms. The upstream manufacturer can either commit to a *restricted mechanism*, $QF \equiv \left\{q(\hat{\theta}), t(\hat{\theta})\right\}_{\hat{\theta}\in\Theta}$, where $q(\hat{\theta})$ and $t(\hat{\theta})$ define a quantity and a franchise fee schedules, respectively. Or, alternatively, she might propose an *unrestricted mechanism*, $RPM \equiv \left\{p(\hat{\theta}), q(\hat{\theta}), t(\hat{\theta})\right\}_{\hat{\theta}\in\Theta}$, which specifies also a retail price target besides a quantity and a franchise fee. With a little abuse of language we shall label the former mechanism as a *"quantity fixing"* contract, whereas the latter one will be referred to as *"resale price maintenance"*. Under this latter contractual regime, the retail price is (costlessly) verifiable by the manufacturer and by a Court enforcing contracts.[19]

[15] See Fudenberg and Tirole (1998, pp. 257) and Laffont and Tirole (1993, pp. 64).

[16] This is just a normalization.

[17] See Laffont and Martimort (2002, Chapter 2) among others.

[18] In order to make use of standard optimization techniques, we shall only consider the class of piecewise differentiable direct revelation mechanisms.

[19] In a related environment Maskin and Riley (1985) and Khalil and Lawarée (1995) examine the choice of output versus input monitoring instrument in a principal-agent relationship.

Noteworthy, a QF arrangement is equivalent to a *vertically decentralized* organizational structure, or, in a more general sense, to an incomplete contract. Under this contractual scheme, the upstream manufacturer does not have enough instruments to monitor the promotional effort level exerted by the retailer. Instead, RPM will be seen to replicate the constrained *vertical integration*[20] outcome, since, by dictating the retail price and the quantity sold to the retailer, the upstream manufacturer is able to *control directly*[21] the retailer's effort level. In this respect, one can think of RPM as being a complete contract.

Timing. Once a contracting mode, either Q or RPM is chosen, the game unfolds as follows:

- **At time t=1.** $\tilde{\theta}$ realizes and only the retailer observes it.

- **At time t=2.** The upstream monopolist makes a take-it-or-leave-it offer to the retailer according to the chosen contracting mode.

- **At time t=3.** The retailer accepts or refuses this offer. If he accepts, he exerts effort e, production occurs and, finally, payments are made according to the contract selected at time t=1. If he rejects, both firms get their outside options which are normalized to zero for simplicity.[22]

3 The Complete Information Benchmark

When the demand parameter $\tilde{\theta}$ is common knowledge and the upstream manufacturer can enforce a (type-dependent) franchise fee, the efficient vertical integration outcome is achieved under both contractual regimes. In both cases, there is full extraction of the retailer's surplus and no double marginalization.

Resale Price Maintenance. Let $t(\theta)$ define the franchise fee paid by the retailer to the manufacturer in order to obtain $q(\theta)$ units of intermediate input. For any given admissible θ, the upstream manufacturer must design an allocation $\left\{q^R(\theta), p^R(\theta), t^R(\theta)\right\}_{\theta\in\Theta}$ which maximizes the franchise fee he gets subject to $e = p + q - \theta$ for all (p, q, θ), and the following retailer's participation constraint:

$$(PC) \quad u(\theta) = p(\theta)q(\theta) - \psi\left(p(\theta) + q(\theta) - \theta\right) - t(\theta) \geq 0.$$

[20] See Tirole (1988, Chapter 4) for a more detailed discussion.

[21] Indeed, for any given pair $\{p(\theta), q(\theta)\}_{\theta\in\Theta}$ chosen by the upstream producer, the optimal effort schedule is directly fixed through the inverse demand function, i.e., $e(\theta) = p(\theta) + q(\theta) - \theta$ for all θ.

[22] Acconcia, Martina and Piccolo (2005) analyze the welfare effects of RPM in the case of type-dependent outside options.

Given Assumption **A2**, the first-order conditions with respect to p and q are necessary and sufficient for optimality:

$$p^R(\theta) = q^R(\theta) = \psi'(e^R(\theta)). \tag{1}$$

At the optimum, marginal revenues are equal to marginal costs. Since by increasing both the retail price and the quantity, the effort must raise too in order to satisfy the identity $e^R(\theta) = 2q^R(\theta) - \theta$ for all θ, the above first-order conditions are readily interpreted.

Quantity Fixing. The retail price is no longer controlled by the manufacturer and the effort maximizes the retailer's profit. For any given θ, the manufacturer's maximizes the franchise fee subject to a standard participation constraint (PC) which must now take into account that the retailer will chose effort optimally.

Formally, for any given admissible θ, the upstream manufacturer must design an allocation $\left\{q^Q(\theta), t^Q(\theta)\right\}_{\theta\in\Theta}$ to maximize $t(\theta)$ subject to:

$$(PC) \quad u(\theta) = -t(\theta) + \max_{e\in\Re_+}\left\{(\theta + e - q(\theta))\, q(\theta) - \psi(e)\right\} \geq 0.$$

Note that the participation constraint (PC) must now take into account that the retailer chooses an effort $e(\theta)$ such that $\psi'(e(\theta)) = q(\theta)$.

Let $\phi(.)$ denote the inverse of $\psi'(.)$. The optimal quantity $q^Q(\theta)$ is defined by the following necessary and sufficient first-order condition:[23]

$$\theta + \phi\left(q^Q(\theta)\right) - 2q^Q(\theta) = 0. \tag{2}$$

Indeed, as both productive technologies (resp. upstream and downstream) operate without costs, this latter condition just says that optimality requires to equalize marginal revenues to zero.

Welfare Analysis. The next proposition whose proof is straightforward shows that, under complete information and when a franchise fee is enforceable, RPM and QF yield exactly the same outcome.

Proposition 1 *Efforts, outputs and fixed-fees are the same under RPM and QF: (i)* $q^R(\theta) = q^Q(\theta) = q^*(\theta)$*; (ii)* $e^R(\theta) = e^Q(\theta) = e^*(\theta)$*; and (iii)* $t^R(\theta) = t^Q(\theta)$*, for all* θ*.*

Even though the retailer's effort is nonverifiable, a franchise fee suffices to achieve the efficient vertical integration outcome under complete information whatever the contractual

[23] Again Assumption **A2** ensures concavity of the manufacturer's objective function.

mode chosen. In both cases, the downstream firm earns zero rents and there is no double marginalization. This result is due to the absence of any vertical externality between the manufacturer and the retailer; both agree on how to set the retailer's effort to increase demand.

As an immediate corollary of Proposition 1, consumers' welfare and manufacturer's profits do not change across contractual modes. Therefore, RPM and QF both achieve exactly the first-best level of profit for the vertically integrated structure.[24]In the next section, however, we prove that under asymmetric information conclusions change drastically because the non-verifiability of effort introduces under adverse selection a vertical externality between the manufacturer and the retailer who no longer agree on the level of this effort.

4 Asymmetric Information

Under both adverse selection and moral hazard, the downstream retailer extracts some information rent. This rent of course depends on whether QF or RPM is the chosen contractual mode.

More specifically, the possibility for the retailer to claim that large sales are due to a high effort level, whereas they result instead from a high demand, induces the upstream manufacturer to give up some information rent to the high demand retailer (high θ) in order to induce truthtelling. As a result, the second-best allocation is characterized by a downward distortion of both quantity and effort supplied by the retailer when he faces a low demand. This effect, in turn, forces the upstream manufacturer to produce above its marginal costs, which generates a double marginalization.

In what follows, we characterize the optimal allocation $(p(\theta), e(\theta), q(\theta))$ for each realization of θ under both RPM and QF. Then, we examine the properties of these allocations and show that: (i) effort is always larger under QF than under RPM; (ii) the impact on quantity is ambiguous.

Resale Price Maintenance. The upstream manufacturer observes and can contract also the retail market price besides the quantity supplied by the downstream firm to final consumers.

The effort level is then indirectly fixed as a function of θ through the inverse demand, i.e., $e = p + q - \theta$ for all (p, q, θ). RPM is *less flexible* than QF. Indeed, whenever the retailer faces a retail price and an input targets, she is indirectly forced to choose the

[24]This is of course a standard result in the IO literature (see Tirole (1988, Chapter 4)).

effort level.[25] In this respect, one can thus think of RPM as being a complete contract since the upstream manufacturer uses all available instruments to control the retailer's activity.

Under asymmetric information, the manufacturer's problem is to design a menu of contracts to maximize the expected franchise fee he receives subject to the retailer's participation and incentive compatibility constraints, together with the additional restriction required by the retail price targets. Given that $e(\theta) = p(\theta) + q(\theta) - \theta$ for all $(p(\theta), q(\theta), \theta)$, the optimal allocation must solve the following maximization program:[26]

$$\mathcal{P}^R \begin{cases} \max_{\langle p(.), q(.), t(.)\rangle} \int_{\underline{\theta}}^{\bar{\theta}} t(\theta)\, dF(\theta) \\ s.t. \\ (IC_1) \;\; \dot{u}(\theta) \; = \psi'(p(\theta) + q(\theta) - \theta) \\ (IC_2) \;\; \dot{q}(\theta) + \dot{p}(\theta) \geq 0 \\ (PC) \;\; u(\theta) = p(\theta)q(\theta) - \psi(p(\theta) + q(\theta) - \theta) - t(\theta) \geq 0. \end{cases}$$

(IC_1) and (IC_2) are respectively the first-order and second-order local conditions for incentive compatibility constraints.[27] The participation constraint (PC) is defined as in the previous section.

Integrating (IC_1) yields the following expression for the information rent of a type-θ retailer:

$$u(\theta) = u(\underline{\theta}) + \int_{\underline{\theta}}^{\theta} \psi'(e(s))ds. \tag{3}$$

High-type retailers have an incentive to mimic low-type ones. Indeed, by misrepresenting this way, retailers earn some information rent since demand is higher than reported, and the effort can be reduced so as to still meet the retail price and quantity targets. This is the intuitive reason why information rents must increase with the effort level.

Before proceeding, let us state a condition which will turn to be useful in what follows.

A4. $\psi''(e) + \psi'''(e)h(\theta) \geq 1/2$ *for all* $(e, \theta) \in \mathbb{R}_+ \times [\underline{\theta}, \bar{\theta}]$.

This is a technical assumption which ensures that the next auxiliary program remains

[25] In a more general sense one can notice that the direct revelation mechanism $\{p(\theta), q(\theta), t(\theta)\}_{\theta \in \Theta}$ resembles the cost reimbursement contract studied in the context of regulation by Laffont and Tirole (1986). In their framework, costs observability plays the same role as price restrictions in ours.

[26] The proof is standard and is thus omitted.

[27] As standard in this literature we prove in the Appendix that those constraints replace the global incentive compatibility constraint.

always concave and displays a unique maximum[28] even if the third derivative of the function $\psi(e)$ might be negative. By using a standard optimization technique, we first solve a properly defined auxiliary program that neglects constraint (IC_2). Then we prove in the Appendix that its solution also solves $\mathcal{P}^R$ since it satisfies both the monotonicity condition (IC_2) and the global incentive compatibility constraint. By using equation (3) together with constraint (PC), and integrating by parts, one obtains the next auxiliary maximization program:

$$\max_{\langle q(.),p(.)\rangle} \int_{\underline{\theta}}^{\bar{\theta}} \left\{p(\theta)q(\theta) - \psi\left(p(\theta)+q(\theta)-\theta\right) - h(\theta)\psi'(p(\theta)+q(\theta)-\theta)\right\} dF\left(\theta\right). \quad (4)$$

Let $\left\{p^R(\theta), q^R(\theta)\right\}_{\theta\in\Theta}$ define the solution to (4). Pointwise optimization yields the following necessary and sufficient conditions:

$$p^R(\theta) - \psi'\left(e^R(\theta)\right) - h(\theta)\psi''\left(e^R(\theta)\right) = 0, \quad (5)$$

$$q^R(\theta) - \psi'\left(e^R(\theta)\right) - h(\theta)\psi''\left(e^R(\theta)\right) = 0. \quad (6)$$

Conditions (5) and (6) together imply that $p^R(\theta) = q^R(\theta)$ for all θ and thus

$$\theta + e^R(\theta) - 2q^R(\theta) = 0. \quad (7)$$

Although the manufacturer has full control of both retail price and sales, she is not able to achieve the first-best because of asymmetric information. Intuitively, when choosing both the optimal retail price and the sales level, the upstream firm must trade-off the (positive) effect on revenues with the (negative) impact of these variables on the retailer's information rent. Since (7) holds, one can notice that whenever p and q rise, e must increase too. Since mimicking is more profitable for the retailer whenever a (sufficiently) high promotional effort level is required, it follows that both the optimal price target and the sales level must entail a downward distortion with respect to the first-best under asymmetric information. Formally, one can easily prove that, under assumptions **A1-A4**, the allocation $\left\{e^R(\theta), q^R(\theta)\right\}_{\theta\in\Theta}$ exhibits downward distortion of both effort and sales, i.e., $e^R\left(\theta\right) \leq e^*\left(\theta\right)$ and $q^R\left(\theta\right) \leq q^*\left(\theta\right)$ for all θ with equality only at $\bar{\theta}$.

Importantly, it should be noticed that: (i) for any given effort level, equation (7) shows that output is set at its monopoly level; and (ii) equation (6) shows that effort is downward distorted to reduce information rent. This is reminiscent of the "dichotomy"

[28] A simple example is provided by the case where $\psi\left(e\right) = (e^2/4) + \alpha \int_0^e \log\left(x+1\right) dx$ with $\alpha \geq 0$ and $F(\theta) = (\theta - \underline{\theta})/(\bar{\theta} - \underline{\theta})$ with $\bar{\theta} - \underline{\theta} \leq 1$. Moreover the additional restriction $\underline{\theta} > 1 + 2\alpha$ also implies interior solutions.

result in Laffont and Tirole (1993, Chapter 3). Quantities are not used for rent extraction purposes: the "monopoly condition" $p(\theta) = q(\theta)$ for all θ already found under complete information still prevails. Of course, because effort is downward distorted the quantity itself falls below its first-best level but this effect is indirect only.

In accordance with the Chicago School, our results highlight that RPM allows the upstream manufacturer to force the retailer to supply the constrained-efficient vertical integration effort level. In this sense, one may conclude that, in our vertical contracting environment, retail price restrictions may help to enforce a *stricter monitoring* regime for those (nonverifiable) non-market activities supplied by retailers.

Implementation of RPM. We conclude the analysis of RPM by proving that the optimal direct revelation mechanism characterized above can easily be implemented with two simple instruments: (i) a menu of linear contracts, and (ii) a retail price target. Instead of using the truthful direct revelation mechanism $\left\{q^R(\theta), p^R(\theta), t^R(\theta)\right\}_{\theta\in\Theta}$, the upstream manufacturer might as well give up any direct communication with the downstream retailer and let him choose the input level within a menu of linear contracts cum RPM, namely $\{T(q, q_0),\ p = q\}$

Corollary 2 *There exists a menu of linear contracts cum RPM* $\{T(q, q_0),\ p = q\}$ *where* $T(q, q_0) = T^R(q_0) + (T^R)'(q_0)(q - q_0)$ *that implements the same allocation as the optimal direct revelation mechanism* $\left\{p^R(\theta), q^R(\theta), t^R(\theta)\right\}_{\theta\in\Theta}$.

This indirect mechanism exhibits both retail price restrictions and *productivity rewards*, which are commonly observed in several forms of *vertical integration*. Formally, the possibility of implementing the optimum with a linear contract cum RPM is simply due to the convexity of the monetary transfer paid by the manufacturer to the downstream firm. When this property holds, one can replace the nonlinear contract $T^R(q) = t^R(\theta^R(q)))$ (if $p = q$ and $+\infty$ otherwise) obtained from $t^R(\theta)$ and $q^R(\theta)$[29] by the menu of its linear tangents. Accordingly the retailer chooses within this menu his most preferred linear contract, $T(\cdot, q_0)$, and then the level of input purchased q. Under RPM, the retail price is fixed to that output level to maintain the monopoly optimality conditions even though, under adverse selection, the retailer 's effort is no longer first-best.[30]

Quantity Fixing. The upstream manufacturer commits now *not* to observe the ex-post realization of the retail price, but she still can observe and contract on the market

[29]Using nonlinear instruments instead of direct revelation mechanism is sometimes referred to as using the *Taxation Principle*. See Laffont and Martimort (2002, pp. 375-379) and Rogerson (1998) for a more detailed discussion.

[30]One can easily check that in this set-up the same result can be obtained also with a price floor, i.e., $p \geq q$, or with a price ceiling, i.e., $p \leq q$.

quantity supplied by the downstream firm on the retail market.[31] A QF mechanism can thus be viewed as an incomplete contract relative to RPM since the upstream producer gives up a screening instrument.

The manufacturer's problem is to design a menu of contracts $\{q(\theta), t(\theta)\}_{\theta\in\Theta}$ to maximize the expected franchise fee subject to participation and incentive compatibility constraints for all θ. By using standard techniques the maximization program may be written as:[32]

$$\mathcal{P}^Q \begin{cases} \max\limits_{\langle q(.),t(.)\rangle} \int_{\underline{\theta}}^{\overline{\theta}} t(\theta)\, dF(\theta) \\ s.t. \\ (IC_1)\ \ \dot{u}(\theta) = q(\theta) \\ (IC_2)\ \ \dot{q}(\theta) \geq 0 \\ (PC)\ \ u(\theta) = -t(\theta) + \max\limits_{e\in\Re_+} \{(\theta + e - q(\theta))\, q(\theta) - \psi(e)\} \geq 0. \end{cases}$$

Importantly, the level of sales is now the only screening device available to the manufacturer. For any given quantity schedule specified by the direct revelation mechanism QF, the downstream retailer gains *flexibility* under a quantity-fixing arrangement in the sense that the effort level is chosen to command more information rents than it would be efficient from the manufacturer's viewpoint. This form of contractual incompleteness, in fact, provides the retailer with *some monopoly power*. More specifically, while choosing the optimal effort level, the retailer does not internalize the impact of his effort on the information rent given up by the upstream manufacturer. QF introduces a kind of vertical externality between the manufacturer and his retailer which was absent under complete information. As rents and effort are positively related via quantity, it will be thus profitable to *oversupply* effort relative to RPM everything else being kept equal.

Integrating (IC_1), one gets the following expression of the information rent of a type-θ retailer:

$$u(\theta) = u(\underline{\theta}) + \int_{\underline{\theta}}^{\theta} q(s)ds. \tag{8}$$

Since effort is set so that to maximize the retailer's profit, i.e., $q(\theta) = \psi'(e(\theta))$, it is immediate to see that a higher effort increases also this information rent.

Below we first solve an auxiliary maximization program which neglects constraint (IC_2). Then, we prove in the Appendix that its solution optimizes also program $\mathcal{P}^Q$ as it satisfies both the monotonicity condition (IC_2) and the global incentive compatibility

[31] This is a simplifying assumption ruling out the possibility of input storing. In our model, all the units of input supplied by the upstream producer to the downstream retailer are sold on the final market. See Section 6 below which shows the robustness of our result.

[32] Again, the proof is standard and is omitted.

constraint. By using (8) together with the participation constraint (PC), and integrating by parts, one obtains thus the next auxiliary program:

$$\max_{\langle q(.)\rangle} \int_{\underline{\theta}}^{\overline{\theta}} \left\{ \left(\theta + \phi\left(q(\theta)\right) - q(\theta)\right) q(\theta) - h(\theta)q(\theta) - \psi\left(\phi\left(q(\theta)\right)\right) \right\} dF\left(\theta\right). \tag{9}$$

Pointwise optimization of the integrand in (9) yields the necessary and sufficient first-order condition for the optimal output $q^Q(\theta)$:[33]

$$\theta + \phi\left(q^Q(\theta)\right) - 2q^Q(\theta) - h(\theta) = 0. \tag{10}$$

Equation (10) shows that the upstream manufacturer chooses the optimal quantity so as to equalize her virtual marginal revenues to zero. However, under asymmetric information and QF, the demand parameter θ has now to be replaced by a lower virtual demand parameter $\theta - h(\theta)$. By doing so, output is reduced below the first-best. The allocation of a low-demand type becomes less attractive to a high-demand one and the latter's incentives to hide his type are mitigated. Reducing the output of an agent with type θ^* reduces the information rents left to higher types $\theta \geq \theta^*$. This explains intuitively why larger hazard rates lead to lower virtual demand and to a stronger double marginalization effect.

Under Assumption **A2**, equation (10) implies that the optimal allocation exhibits downward distortions on both quantity and effort, i.e., $q^Q\left(\theta\right) \leq q^*\left(\theta\right)$ and $e^Q(\theta) \leq e^*(\theta)$ for all θ with equality only at $\overline{\theta}$. To minimize the information rents granted to high types, the upstream manufacturer must give up productive efficiency by distorting downward the input supplied to low types.

Implementation of QF. The optimal direct revelation mechanism characterized above can be implemented by a simple menu of linear contracts. Instead of using the truthful direct revelation mechanism $\left\{q^Q(\theta), t^Q(\theta)\right\}_{\theta\in\Theta}$, the upstream manufacturer might let the downstream retailer choose the input level within a menu of linear contracts which are tangent to a properly defined non-linear schedule $T^Q(q) = t^Q(\theta^Q(q))$ obtained from $t^Q(\theta)$ and $q^Q(\theta)$. The next corollary proves formally the result.

Corollary 3 *There exists a menu of linear contracts of the form* $T(q, q_0) = T^Q(q_0) + (T^Q)'(q_0)(q - q_0)$ *that implements the same allocation as the optimal direct revelation mechanism* $\left\{t^Q(\theta), q^Q(\theta)\right\}_{\theta\in\Theta}$.

[33]It is immediately to see that whenever Assumption **A2** is satisfied the first-order condition (10) is also sufficient for a global optimum.

The menu of linear contracts $T(q, q_0)$ has three main features. First, it entails a franchise fee; second it involves a linear (price) component; and, finally it also exhibits some form of quantity discounts[34] which are commonly observed in some manufacturers-retailers relationships. As under RPM, this last property is again simply due to the concavity of the non-linear schedule $T^Q(q)$, so that the (average) "price" paid to obtain q units of intermediate input is decreasing with respect to the amount bought. Then, one can replace the nonlinear contract $T^Q(q)$ by the menu of its linear tangents.

RPM versus QF. Let us now compare the allocations obtained under RPM and QF. While the results obtained for effort schedules seem to confirm the findings of previous contributions, strikingly, the conclusions achieved for the optimal quantity schedules may not.

For the ease of the presentation, let consider first a cutting-edge case where the marginal disutlity of effort is linear, i.e., $\psi(e) = e^2/2$. Both types of contracts entail then the same output, $q^Q(\theta) = q^R(\theta) = \theta - h(\theta)$ for all θ. However, under QF, the agent exerts more effort relative to RPM, $e^Q(\theta) \geq e^R(\theta)$ for all θ with equality only at $\overline{\theta}$.

More generally, two effects are at play simultaneously once one moves from the complete (RPM) to the incomplete contracting environment (QF):

• First, for a given quantity, the agent exerts more effort under QF relative to RPM. Under QF, the agent is residual claimant for the full impact of his effort on enhancing demand. This effect raises effort and thus production: a *"demand-enhancing effect".*

• Second, given that sales is the only screening instrument under QF, one needs to distort it downward for rent extraction reasons: a *"rent extraction effect".* The dichotomy result between production and incentives no longer holds.

To better understand these two effects, it is useful to represent graphically on Figure 1 below outputs and efforts under both QF and RPM.

[34] This is akin to the quantity discount result due to Maskin and Riley (1984).

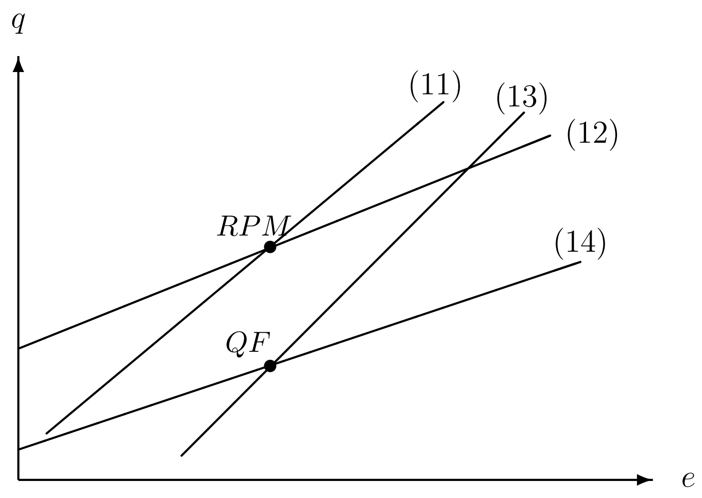

Figure 1

On Figure 1, we observe that the RPM allocation is obtained at the intersection of:

$$q = \psi'(e) + h(\theta)\psi''(e) \tag{11}$$

and

$$q = \frac{\theta + e}{2}. \tag{12}$$

The QF allocation is instead obtained at the intersection of:

$$q = \psi'(e) \tag{13}$$

and

$$q = \frac{\theta - h(\theta) + e}{2}. \tag{14}$$

The fact that (13) lies below (11) captures the demand-enhancing effect which pushes effort and output up. That (14) lies below (12) represents the rent-extraction effect. It reduces production. These two effects exactly compensate each other in the case of a quadratic disutility function, $\psi'''(.) = 0$. The proposition below states formally the result in a more general setting.

Proposition 4 *Assume that **A1-A4** hold altogether, the allocations* $\left\{e^Q(\theta), q^Q(\theta)\right\}_{\theta\in\Theta}$ *and* $\left\{e^R(\theta), q^R(\theta)\right\}_{\theta\in\Theta}$ *satisfy the following properties: (i)* $e^Q(\theta) \geq e^R(\theta)$ *for all* θ *(with equality only at* $\bar{\theta}$*); and (ii)* $q^R(\theta) \geq q^Q(\theta)$ *(resp.* ≤ 0*) for all* θ *(with equality only at* $\bar{\theta}$*) if* $\psi'''(e) \geq 0$ *(resp.* ≤ 0*) for all* $e \geq 0$*.*

Part (i) of Proposition 4 has a simple economic interpretation. When a QF contract is enforced, the upstream manufacturer gives up the control of the retailer's effort. This decentralized organizational mode, crucially, allows the downstream firm to secure more information rents by playing on his effort choice. More specifically, as effort affects positively quantity,[35] the retailer profitably supplies more effort (relative to RPM) in order to enjoy more rents.

Part (ii) reveals an important point. As we have pointed out before, by dictating both the retail price and the sales level, the upstream firm *mitigates* the retailer's incentives to misrepresent his type. Although under RPM the retailer's incentives to increase his information rent by exerting a higher effort level are reduced, asymmetric information still limits the possibility of achieving the first-best. Hence, a downward distortion of the optimal allocation is still needed.[36] In this respect, the above proposition provides conditions under which RPM enhances (resp. weakens) *productive efficiency* with respect to QF.

The trade-off between the *enhancing-demand effect* and the *rent extraction effect* crucially depends on the third derivative of the effort disutility $\psi(e)$. Whenever $\psi'''(.) > 0$ (resp. < 0) the latter effect dominates (resp. is dominated by) the former one. To understand the respective magnitude of these effects, it is useful to rewrite the derivative of the information rents under both QF and RPM if the same output schedule $q(\theta)$ were implemented. With obvious notations, we would have:

$$\dot{u}^Q(\theta) = q(\theta) \text{ and } \dot{u}^R(\theta) = -\psi'(e(\theta)) = q(\theta) - h(\theta)\psi''(2q(\theta) - \theta)$$

where the last equality follows by using (11) and (12). These equations immediately highlight that a greater output distortion is needed to reduce the agent's rent under RPM than under QF whenever $\psi'''(.) < 0$.

5 Welfare Analysis

In light of the previous results, it becomes natural to examine how the two kinds of contractual modes affect both consumers' welfare and profits. As standard in the Antitrust literature, our main welfare measure will be consumers' surplus.[37]

Before proceeding, recall that both types of contracts entail a double marginalization because of the additional costs due to the information rent of the downstream retailer. It follows that, in order to evaluate whether retail price restrictions are detrimental to

[35] Remember that under QF the downstream dealer chooses his effort according to the rule $q = \psi'(e)$.

[36] This distinguishes our analysis from Gal-Or (1991) where RPM is a sufficient instrument to nullifies information rents when the only piece of private information held by the retailer is on demand.

[37] Much scholars have indeed advocated that the sole role of Antitrust policies should be to promote consumers' surplus. See Bork (1978, Chapter 2, pp. 51) for instance.

consumers' welfare, one needs to examine which contractual mode entails the largest double mark-up effect, i.e., the largest quantity distortion.

Let $E_\theta[\Delta CS(\theta)] \equiv E_\theta[CS^R(\theta)] - E_\theta[CS^Q(\theta)]$ define the difference between the expected consumers' surplus under RPM and QF, respectively.

Proposition 5 *Assume A1-A4, the following properties hold: (i)* $E_\theta[\Delta CS(\theta)] \geq 0$ *(resp.* ≤ 0*) if* $\psi'''(\cdot) \geq 0$ *(resp.* ≤ 0*); and (ii) the upstream manufacturer always prefers RPM to QF.*

As in Gal-Or (1991), RPM increases consumers' welfare only when it improves upon productive efficiency with respect to QF. By introducing moral hazard, our analysis emphasizes also that this result no longer holds when $\psi'''(\cdot) < 0$. In this case, the adoption of retail price restrictions, aimed at solving the agency problem, induces a (social) cost in terms of productive efficiency which is too large with respect to QF.[38]

Finally, part (ii) has a natural economic interpretation. Since the set of *(screening) instruments* available to the upstream manufacturer under RPM is larger than that offered by QF, the upstream manufacturer must prefer RPM to QF.

Only when the marginal disutility of the retailer's effort is convex, it is the case that RPM, which is the most preferred contracting mode for the manufacturer from an ex ante viewpoint, also improves consumers' surplus. From an operational viewpoint, Antitrust authorities may thus find it a priori quite difficult to conclude on the impact of RPM without having some specific knowledge of the retailers' cost of services.

This mixed conclusion may not be of much practical value for Antitrust authorities. Fine details on the shape of the disutility function may be hard to ascertain a priori. Alternatively, we will see below that a more operational criterion is available if the industry chooses the contractual mode to maximize ex ante joint-profit.

From an ex ante viewpoint, the contractual mode which maximizes the joint-profit of the vertical structure solves:

$$\max_{\omega \in \mathcal{M}} E_\theta \left[R\left(e^\omega(\theta), q^\omega(\theta), \theta\right) - \psi\left(e^\omega(\theta)\right) \right],$$

where $R\left(e^\omega(\theta), q^\omega(\theta), \theta\right) \equiv (\theta + e^\omega(\theta) - q^\omega(\theta))q^\omega(\theta)$ defines the revenue function for all θ and $\omega \in \mathcal{M}$. Let us define by $E_\theta[\Delta\Pi(\theta)]$ the difference between the joint profits of the vertical structure under RPM and QF respectively.

The next proposition highlights the crucial role played by the relative magnitude of the distortion in *productive* and *allocative*[39] efficiency involved by QF and RPM, respectively.

[38] See also Comanor (1985).

[39] We refer to productive efficiency to measure the distance between the complete information sales

Proposition 6 *The joint-profit of the vertical structure is greater (resp. lower) with RPM than with QF if* $\psi'''(\cdot) > 0$ *(resp.* ≤ 0*).*

This proposition has a clear economic interpretation. The vertical structure would like to commit ex ante to the contract minimizing distortion with respect to the complete information benchmark.

Two cases may occur depending on the sign of $\psi'''(\cdot)$:

- *Case 1:* $q^R(\theta) \leq q^Q(\theta)$ and $e^Q(\theta) \geq e^R(\theta)$, for all θ. Then, QF is preferred to RPM by the vertical structure. Intuitively, by refusing to enforce retail price restrictions, the manufacturer/retailer coalition benefits from two effects: first an improvement in productive efficiency, i.e., $q^Q(\theta) \geq q^R(\theta)$ for all θ; second an increase in the consumers' willingness to pay for the final product, i.e., the enhancing-demand effect $e^Q(\theta) \geq e^R(\theta)$ for all θ.

- *Case 2:* $q^R(\theta) \geq q^Q(\theta)$ and $e^Q(\theta) \geq e^R(\theta)$, for all θ. The coalition must now trade off the rent-extraction effect (productive efficiency) with the enhancing-demand effect (allocative efficiency). However, we show in the Appendix that the productive efficiency gain involved by RPM always overcomes the effect of surplus extraction due to the possibility of facing consumers with a higher willingness to pay produced under QF.

In practice, variations in the sign of $\psi'''(\cdot)$ may capture several natural scenarios. For instance, the case where $\psi'''(\cdot) < 0$, which implies that the marginal costs of effort are increasing and concave, seems to fit the idea that there are some *learning-by-doing economies* in the production of effort. Meaning that the larger is the exerted effort level the lower are its associated marginal costs. This could well happen when retailers produce indivisible services whose production benefits from specific (marketing) skills which can be acquired only over the time.[40] On the other hand, the case where $\psi'''(\cdot) > 0$ captures the situation where marginal costs of effort are increasing and convex. This scenario seems to fit well "pure advertising", where the marginal costs of a brand entering a new market are convex because of localization and transportation costs.

The previous result has an immediate but important corollary which reveals a policy implication supporting, to some extent, the Chicago-school conjecture.

level and those obtained under either QF or RPM. Instead, we refer to allocative efficiency to measure the difference between the complete information effort level and those obtained either under RPM or QF.

[40]Of course, this would certainly require modelling the relationship between the manufacturer and his retailer in a dynamic setting. However, one could view our static model as a convenient short cut capturing this effect when ψ''' is negative.

Corollary 7 *Whenever RPM is ex ante optimal from the vertical structure's viewpoint, it also raises consumers' surplus and total welfare relative to QF.*

Since consumers prefer RPM when it increases productive efficiency relative to QF, and the vertical structure commits to RPM only if it raises *sufficiently* productive efficiency with respect to QF, it can be easily concluded that the use of price restrictions always generates a Pareto improvement relative to QF.

In some sense, this result extends the traditional Chicago view to a more sophisticated environment where firms use more complex strategies than linear prices.[41] The main policy message of the section reveals that, whenever vertical structures choose contractual modes according to an ex ante criterion of joint-profit maximization, the choice of price restrictions must be regarded as optimal. Remarkably, it follows that, by *banning* the practice of vertical price fixing, an antitrust authority might harm consumers and decrease total welfare in some cases.

Given this result, the true underlying issue is whether ex ante joint-profit maximization for the choice of contractual mode is plausible both from a theoretical and a practical viewpoints.

One may actually think of several justification for joint-profit maximization as the relevant criterion to judge organizational choices. Indeed, there is no reason to think that the same allocation of bargaining powers between the manufacturer and the retailer prevails ex ante and ex post.[42] Ex post, once the retailer knows his private information on demand, the investment in promotional effort may be quite specific and not easily redeployable towards other uses. As a result, the retailer has no bargaining power ex post. On the contrary, ex ante, i.e., before the retailer learns about the demand parameter and before he makes any specific investment towards improving promotional services on this particular market, the retailer keeps a more equal bargaining power with the manufacturer and joint-profit maximization becomes quite relevant as an ex ante criterion. Once this ex ante choice is made, the manufacturer and the retailer can share ex ante joint-profit through a lump-sum fixed-fee which yields a (non type-dependent) reservation value to the retailer.[43] Second, one should not expect vertical structures which do not maximize

[41] In particular, it can be seen as a modified version of Bork's argument if favour of vertical price fixing.

[42] Actually, this shift in bargaining powers is rather standard in the incomplete contract literature which assumes that parties have both equal bargaining powers ex post, once some non-verifiable variables become publicly observable, but ex ante organizational choices are made according to an efficiency criterion. See Laffont and Martimort (2002, Chapter 6) for some remarks on this. The same perspective can be taken here but in an asymmetric information framework.

[43] Of course, in a full fledged model, one may want to model explicitly this two-stage bargaining process. The fact that the first-stage lump-sum transfer do not depend on the retailer's type by definition would not invalidate our analysis of the ex post bargaining game since the corresponding reservation payoff of

ex ante joint-profit to survive in the long-run if we were modelling explicitly entry on both sides of the market.

In practice, sunk investments on the retailer's side and free entry both in manufacturing and retailing are two easily checkable conditions that may be used in practice by antitrust authorities to ascertain the optimality of RPM.

6 Partially Verifiable Output

We now briefly extend our analysis to the case where the upstream producer no longer controls the level of output sold in the market by the retailer. One must now take into account that the retailer might decide to sell on the downstream market only a fraction of the quantity bought from the manufacturer. We shall prove that this loss of control entails no real cost for the manufacturer.

Of course, this extra freedom is only relevant in the case of a RPM mechanism.[44] Under RPM, the manufacturer's program must now be more carefully stated. In particular, while the direct, truthful revelation mechanism still specifies a transfer, $t(\hat{\theta})$, a retail price level, $p(\hat{\theta})$, and a quantity sold to the retailer, $q(\hat{\theta})$, for each retailer's message $\hat{\theta}$, one must also take into account that the retailer might decide to sell only a fraction of this quantity on the downstream market. Formally, this amounts to consider an additional constraint $q \leq q(\hat{\theta})$ to the producer's and the retailer's programs, where q is now the output actually sold by the retailer on the final market.

Let us then proceed backward by considering the retailer's utility level after the allocation $(t(\hat{\theta}), q(\hat{\theta}), p(\hat{\theta}))$ has been offered given the message $\hat{\theta} \in \Theta$. It is defined as:

$$u(\theta, \hat{\theta}) = \max_{q \leq q(\hat{\theta})} \left\{ p(\hat{\theta})q - \psi(p(\hat{\theta}) + q - \theta) - t(\hat{\theta}) \right\}$$

From the concavity of this objective function with respect to q, the quantity $q^S(\theta)$ that the retailer sells on the downstream market when truthfully reporting his type to his manufacturer is thus $q(\theta)$ whenever:

$$p(\theta) > \psi'(p(\theta) + q(\theta) - \theta) = \psi'(e(\theta)).$$

Now consider the optimal mechanism $(t^R(\hat{\theta}), q^R(\hat{\theta}), p^R(\hat{\theta}))$ that is offered when the

the retailer would be fixed. Beyond this rather stark difference between the allocations of ex ante and ex post bargaining powers, the lessons of our model would carry over to more symmetric distributions of bargaining power.

[44] It can be readily seen that the manufacturer can choose to sell to the retailer a quantity so that the latter does not want to put less on the final downstream market.

downstream output is contractible. From (5), we have:

$$p^R(\theta) > \psi'(p^R(\theta) + q^R(\theta) - \theta) = \psi'(e^R(\theta)).$$

Thus, the retailer has no incentives to sell less on the downstream market than what he buys from his principal. Therefore, $(t^R(\hat{\theta}), q^R(\hat{\theta}), p^R(\hat{\theta}))$ remains an optimal mechanism when the final output is no longer fully verifiable.

The intuition is straightforward. We have already seen that, for screening purposes, the optimal RPM mechanism reduces the quantity below what would be optimal under complete information. Now, consider the output choice of the retailer when this output is non-verifiable. Since the retailer's objectives are similar to those of the vertical structure under complete information, the retailer would like to expand output up to the point where the marginal benefit of one extra unit, i.e., the retail price, is equal to the marginal disutility of effort. Clearly, the retailer would like to expand output above $q^R(\theta)$ and he has no incentives to sell less on the downstream market.

This shows that our results are robust to the lack of verifiability of the final quantity sold by the retailer.

7 Conclusion

We have analyzed the private and the welfare properties of vertical contracts based on retail price restrictions in a successive monopolies framework under both adverse selection and moral hazard. The analysis reveals two main insights.

First, RPM, although offering a complete set of screening instruments to manufacturers and increasing thereby their profit, might be detrimental to consumers since it distorts too much productive efficiency with respect to QF. In this case, we have proved that the contrasted conclusions of respectively the Chicago School advocating for the legitimacy of retail price restrictions, and some more recent theoretical contributions, criticizing the Chicago posture, might both emerge as special cases. Second, when it is preferred by the vertical structure as a whole, RPM also maximizes consumers' surplus and total welfare.

From an antitrust policy perspective, assessing whether RPM should be allowed or not might require knowledge of fine details of the retailers' technology for providing services or much confidence on the fact that long run market structure emerges to maximize the ex ante joint-profit of the manufacturer/retailer coalition. To some extent, these results can be viewed as confirming the Chicago School conjecture in a setting where information asymmetries force upstream firms to use more complex instruments than linear prices.

Some extensions of our analysis would deserve further attention. First, one could generalize our results to other functional forms. A key simplifying feature of our modelling was the fact that some sort of dichotomy holds under RPM. Finding out general arguments on the dominance or not of RPM may not be as clear as soon as such dichotomy no longer holds. Second, the choice of a contracting mode may have some strategic value in the context of competing hierarchies and these issues would be worth investigating. Finally, we have focused on moral hazard at the retailers' level but it would be worth to introduce moral hazard at the level of the manufacturers to have a more complete view of the agency problems introduced by the different vertical restraints. We hope to investigate some of these issues in future research.

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8 Appendix

8.1 Sufficient Conditions for Optimality under RPM

The next lemma shows two technical results. First, it proves that under Assumptions **A1-A4**, conditions (5)-(6) are necessary and sufficient for a global maximum. Second, it verifies that the solution to program (4) also solves $\mathcal{P}^R$.

Lemma 8 *Assume **A1-A4**, the following properties hold: (i) the first-order necessary conditions (5)-(6) are also sufficient for a global optimum; (ii) the solution to program (4) also optimizes $\mathcal{P}^R$.*

(i) For any given θ, let $\mathcal{H}(\theta)$ define the Hessian matrix of the second-order conditions associated to the maximization program (4), with:

$$\mathcal{H}(\theta) = \begin{pmatrix} -\left(\psi''(e) + h(\theta)\psi'''(e)\right) & 1-\left(\psi''(e) + h(\theta)\psi'''(e)\right) \\ 1-\left(\psi''(e) + h(\theta)\psi'''(e)\right) & -\left(\psi''(e) + h(\theta)\psi'''(e)\right). \end{pmatrix}$$

The second-order sufficient conditions for a global maximum requires $\mathcal{H}(\theta)$ being strictly negative defined. Let $\det[\mathcal{H}(\theta)]$ denote the determinant of $\mathcal{H}(\theta)$. We have $\det[\mathcal{H}(\theta)] = 2\left(\psi''(e) + h(\theta)\psi'''(e)\right) - 1 > 0$, when **A4** holds.

(ii) Since the optimality conditions (5)-(6) require $p^R(\theta) = q^R(\theta)$ for all θ, in order to prove the claim it suffices to verify that: (1) the schedule $q^R(\theta)$ satisfies constraint IC_2 for all θ; and (2) the global incentive compatibility constraint holds at the optimal allocation $\left\{p^R(\hat\theta), q^R(\hat\theta), t^R(\hat\theta)\right\}_{\hat\theta\in\Theta}$ for all pairs $(\theta, \hat\theta)$ with $\theta \neq \hat\theta$.

First note that by using (6) and (7) a simple application of the Implicit Function Theorem yields:

$$\dot{q}^R(\theta) = \frac{\psi''(e^R(\theta)) - \psi''(e^R(\theta))\dot{h}(\theta) + h(\theta)\psi'''(e^R(\theta))}{2\left(\psi''(e^R(\theta)) + h(\theta)\psi'''(e^R(\theta))\right) - 1} > 0$$

under Assumptions **A1-A4** which proves immediately the claim. ■

Then recall that in the optimum $p^R(\theta) = q^R(\theta)$ and $e^R(\theta) = 2q^R(\theta) - \theta$ for all θ. Let $u^R(\theta, \hat{\theta})$ define the retailer's profits evaluated at the optimal allocation $\left\{p^R(\hat{\theta}), q^R(\hat{\theta}), t^R(\hat{\theta})\right\}_{\hat{\theta}\in\Theta}$ when the true retailer's type is θ and the message sent to the manufacturer is $\hat{\theta}$, with $\hat{\theta} \neq \theta$. Define $\Gamma(\theta, \hat{\theta}) \equiv u^R(\theta, \theta) - u^R(\theta, \hat{\theta})$, and without loss of generality assume $\theta > \hat{\theta}$. Simple algebraic manipulations allow to rewrite $\Gamma(.)$ as:

$$\Gamma(\theta, \hat{\theta}) = \int_{\hat{\theta}}^{\theta} \left\{2q^R(s)\dot{q}^R(s) - \dot{t}^R(s) - 2\psi'(e^R(\theta, s))\dot{q}^R(s)\right\} ds$$

By using IC_1 and substituting for $\dot{t}^R(s) = \dot{p}^R(s)q^R(s) + p^R(s)\dot{q}^R(s) - \psi'(e^R(s,s))(\dot{p}^R(s) + \dot{q}^R(s))$ into the above equation, one obtains $\Gamma(\theta, \hat{\theta}) = \int_{\hat{\theta}}^{\theta} 2\dot{q}^R(s) \left\{\int_s^{\theta} \psi''(e^R(x,s))dx\right\} ds$. Since Assumption **A2** implies that $\psi''(.) > 0$ for all $e \geq 0$, it follows that $\Gamma(\theta, \hat{\theta}) > 0$ which immediately proves the claim. ■

8.2 Proof of Corollary 2

As the direct, deterministic revelation mechanism $\omega^e \equiv \{e(\theta), p(\theta), w(\theta)\}_{\theta\in\Theta}$ replicates the optimal allocation derived under RPM, in order to prove the result one may restrict attention to this contract without loss of generality. The claim is proven in two steps. First, in step 1, we show that the optimal allocation under RPM is implemented by the indirect nonlinear increasing and convex schedule $T^R(q)$ together with the auxiliary constraint defined by the retail price target $p = q$. Then, in step 2, we prove that the contract $\left\{T^R(q), p = q\right\}$ is implemented by a properly defined menu of its linear tangents.

Proof of step 1. As $q^R(\theta)$ is increasing in θ, let $\theta^R(q)$ define its inverse. Assume that the upstream manufacturer offers the indirect mechanism $\left\{T^R(q) \equiv w^R(\theta^R(q)),\ p = q\right\}$ to the retailer. Under this contract the downstream firm receives a fixed monetary transfer $T^R(q)$ for any given selected input level q. Differentiating $T^R(q)$ with respect to q one gets $\dot{T}^R(q) \equiv \dot{w}^R(\theta^R(q))/\dot{q}^R(\theta)$. Moreover, since $q^R(\theta) = p^R(\theta)$ for all θ, by definition of information rents, one can show that:

$$w^R(\theta) - \psi\left(e^R(\theta)\right) = \int_{\underline{\theta}}^{\theta} \psi'(e^R(s))ds$$

where optimality requires $e^R(\theta) = 2q^R(\theta) - \theta$ for all θ. Differentiating the above equation with respect to θ, simple algebra immediately implies that $\dot{T}^R(q) = 2\psi'(e^R(\theta)) \geq 0$ for all q, which is precisely the first-order condition of the retailer's problem when he chooses the input level within the schedule $T^R(q)$. Hence it is straightforward to prove that the indirect mechanism $\left\{T^R(q), p = q\right\}$ implements the allocation $\left\{p^R(\theta), q^R(\theta)\right\}_{\theta\in\Theta}$.

Moreover, differentiating again with respect to q one gets

$$\frac{\ddot{T}^R(q)}{2\psi''(e^R(\theta))} = \frac{(2\psi''(e^R(\theta))|\dot{h}(\theta)|+1)}{\psi''(e^R(\theta))+\psi''(e^R(\theta))|\dot{h}(\theta)|+\psi'''(e^R(\theta))h(\theta)}$$

Finally, by using assumption **A4** and $\psi''(e) > 0$ for all $e \geq 0$, one can show that $\ddot{T}^R(q) \geq 0$ for all q since both the denominator and the numerator in the above equation are positive. ■

Proof of step 2. Consider the menu of linear contracts $T(q, q_0)$ tangent to $T^R(q)$, i.e., such that $T(q, q_0) = T^R(q_0) + \dot{T}^R(q_0)(q - q_0)$. And, assume that the upstream manufacturer proposes a linear contract $\{T(., q_0), \ p = q\}$. The retailer chooses which tangent is its most preferred one (q_0) and the level of input (q). Moreover, the contract also requires the additional constraint dictating a price target such that, for any given q, the retail price[45] must satisfy $p = q$. The optimal pair (q_0^R, q^R) must then be the unique solution to the following optimization program

$$\max_{\langle q, q_0 \rangle} T(q, q_0) - \psi(2q - \theta) \tag{15}$$

The first-order necessary conditions are given then by

$$\begin{aligned} \dot{T}^R\left(q_0^R\right) - 2\psi'(2q^R - \theta) &= 0 \\ -\ddot{T}^R\left(q_0^R\right)\left(q^R - q_0^R\right) &= 0 \end{aligned}$$

Since $T^R(q)$ is a convex function it follows that $q^R = q_0^R = q^R(\theta)$ for all θ. Which proves the claim provided that $q^R(\theta)$ is a global optimum of (15). To check sufficiency one needs simply to verify that the corresponding Hessian $\mathcal{H}^R(\theta)$ of the second-order derivatives must be strictly definite negative at $q^R(\theta)$ for all θ.

$$\mathcal{H}^R(\theta) = \begin{pmatrix} -4\psi''\left(e^R(\theta)\right) & \ddot{T}^R\left(q^R(\theta)\right) \\ \ddot{T}^R\left(q^R(\theta)\right) & -\ddot{T}^R\left(q^R(\theta)\right) \end{pmatrix}$$

As $\psi''(e) > 0$ for all $e \geq 0$ by **A2**, $T^R\left(q^R(\theta)\right)$ is convex and $q^R(\theta)$ is increasing, the claim follows immediately since $\det[\mathcal{H}^R(\theta)] = \ddot{T}^R\left(q^R(\theta)\right)\psi''\left(e^R(\theta)\right)(2 + 1/\dot{q}^R(\theta)) \geq 0$ for all θ. ■

8.3 Sufficient Conditions for Optimality under QF

The next lemma proves formally that solving the auxiliary program (9) is equivalent to solving $\mathcal{P}^Q$.

[45] This mechanism clearly implements the optimal level of effort $e = 2q - \theta$.

Lemma 9 *Assume **A1-A3**, the following properties hold: (i) the first-order necessary condition (10) is also sufficient for a global optimum;(ii) the solution to program (9) also optimizes $\mathcal{P}^Q$.*

(i) The proof follows immediately since $\psi''(e) > 1/2$ for all e by **A2**.

(ii) In order to prove the claim it suffices to verify that: (1) the optimal quantity schedule $q^Q(\theta)$ satisfies IC_2 for all θ; and (2) the global incentive compatibility constraint holds at the optimal allocation $\left\{e^Q(\theta), q^Q(\theta), t^Q(\theta)\right\}_{\theta\in\Theta}$.

First, observe that, by equation (10), a simple application of the Implicit Function Theorem allows to obtain $\dot{q}^Q(\theta) = (1 - \dot{h}(\theta))/(2 - \phi'\left(q^Q\right))$. Since assumptions **A1** and **A3** imply respectively $\dot{h}(\theta) \leq 0$ for all θ and $2 - \phi'(.) > 0$ for all q, the proof follows immediately. ■

Next, define by $u^Q(\theta, \hat{\theta})$ the retailer's utility function evaluated at the optimal allocation $\left\{e^Q(\hat{\theta}), q^Q(\hat{\theta}), t^Q(\hat{\theta})\right\}_{\hat{\theta}\in\Theta}$, whenever the true type is θ and the message sent to the manufacturer is $\hat{\theta}$, with $\hat{\theta} \neq \theta$. Let $\Gamma(\theta, \hat{\theta}) \equiv u^Q(\theta, \theta) - u^Q(\theta, \hat{\theta})$, and without loss of generality assume that $\theta > \hat{\theta}$. Simple algebraic manipulations allow to rewrite $\Gamma(\theta, \hat{\theta})$ as:

$$\Gamma(\theta, \hat{\theta}) = \int_{\hat{\theta}}^{\theta} \left\{\theta\, \dot{q}^Q(x) + \dot{q}^Q(x)\phi\left(q^Q(x)\right) - 2\, \dot{q}^Q(x)q^Q(x) - \dot{t}^Q(x)\right\} dx$$

by using IC_1 and substituting for $\dot{t}^Q(x) = \dot{q}^Q(x)(x + \phi\left(q^Q(x)\right) - 2q^Q(x))$ in the above equation, one immediately obtains that $\Gamma(\theta, \hat{\theta}) = \int_{\hat{\theta}}^{\theta}(\dot{q}^Q(x)(\int_u^{\theta}(s^2/2)\,ds)dx$, which is positive and so proves the claim. ■

8.4 Proof of Corollary 3

The proof is organized in two steps. In step 1, we show that the nonlinear indirect mechanism $T^Q(q) \equiv t^Q\left(\theta^Q(q)\right)$ implements the optimal direct revelation mechanism allocation $\left\{e^Q(\theta), q^Q(\theta)\right\}_{\theta\in\Theta}$. Then, in step 2, we prove that $T^Q(q)$ is implemented by a properly chosen menu of its linear tangents.

Proof of step 1. Once again, to reconstruct the indirect mechanism $T^Q(q)$ from the direct revelation mechanism $\left\{t^Q(\theta), q^Q(\theta)\right\}_{\theta\in\Theta}$ is quite easy. Let $\theta^Q(q)$ define the inverse of $q^Q(\theta)$. Consider the indirect mechanism $T^Q(q) \equiv t^Q\left(\theta^Q(q)\right)$, by definition it follows that $\dot{T}^Q(q) \equiv \dot{t}^Q\left(\theta^Q(q)\right)/\dot{q}^Q(\theta)$; moreover, by construction, equation (8) implies:

$$\left(\theta+\phi\left(q^Q(\theta)\right)-q^Q(\theta)\right)q^Q(\theta)-\psi\left(\phi\left(q^Q(\theta)\right)\right)-t^Q(\theta)=\int_{\underline{\theta}}^{\theta} q^Q(s)ds$$

Differentiating the above equation with respect to θ and using an envelope argument, one immediately gets:

$$\dot{t}^Q(\theta)=\left(\theta+\phi\left(q^Q(\theta)\right)-2q^Q(\theta)\right)\dot{q}^Q(\theta)\,,\ \forall\,\theta\in\Theta$$

By using (10), the above condition implies $\dot{T}^Q(q)=h(\theta)\geq 0$ for all θ, which is precisely the first-order condition of the retailer's problem when he chooses the input level within the schedule $T^Q(q)$. Hence it is straightforward to verify that the nonlinear schedule $T^Q(q)\equiv t^Q\left(\theta^Q(q)\right)$ implements the optimal allocation $\left\{e^Q(\theta),q^Q(\theta)\right\}_{\theta\in\Theta}$. Differentiating $\dot{T}^Q(q)$ again, it follows that $\ddot{T}^Q(q)=-|\dot{h}(\theta)|/\dot{q}^Q(\theta)\leq 0$ for all θ, hence $T^Q(.)$ is concave. ■

Proof of step 2. We now show that there exists a menu of linear contracts $T(q,q_0)$ implementing the optimal direct revelation mechanism allocation $\left\{t^Q(\theta),q^Q(\theta)\right\}_{\theta\in\Theta}$. Assume that the upstream manufacturer offers to the downstream retailer a menu of linear contracts $T(q,q_0)$ tangent to the non-linear schedule $T^Q(q)$. Define this menu by $T(q,q_0)=T^Q(q_0)+\dot{T}^Q(q_0)(q-q_0)$, the retailer will then choose both a tangent q_0 and the level of input q such that:

$$(q^Q,q_0^Q)\in\arg\max_{\langle q,q_0\rangle}\left\{(\theta+\phi(q)-q)\,q-\psi(\phi(q))-T(q,q_0)\right\}. \tag{16}$$

The first-order necessary conditions with respect to q_0 and q are respectively:

$$-\ddot{T}^Q(q_0^Q)(q_0^Q-q^Q)=0,$$
$$\theta+\phi\left(q^Q\right)-2q^Q-\dot{T}^Q(q_0^Q)=0.$$

Since $T^Q(.)$ is concave, one may immediately verify that $q^Q=q_0^Q$. It then follows immediately that $(\theta+\phi(q_0^Q)-2q_0^Q)\dot{q}_0^Q-\dot{t}^Q(\theta)=0$ for all θ. Hence, if the above first-order conditions are also sufficient for an optimum, retailers of type θ choose $q^Q=q_0^Q=q^Q(\theta)$ for all θ. Which proves the claim provided that $q^Q(\theta)$ is a global optimum of program (16). In order to check sufficiency one needs simply to verify that the corresponding Hessian $\mathcal{H}^Q(\theta)$ of second-order derivatives must be strictly definite-negative at $q^Q(\theta)$ for all θ:

$$\mathcal{H}^Q(\theta)=\begin{pmatrix}(\phi'\left(q^Q(\theta)\right)-2) & -\ddot{T}^Q\left(q^Q(\theta)\right)\\ -\ddot{T}^Q\left(q^Q(\theta)\right) & \ddot{T}^Q\left(q^Q(\theta)\right)\end{pmatrix}.$$

Since we have shown above that $T^Q(q)$ is concave, and assumption **A2** implies that $\phi'\left(q^Q(\theta)\right)-2<0$, the result follows immediately as $\det[\mathcal{H}^Q(\theta)]=-\ddot{T}^Q\left(q^Q(\theta)\right)/\dot{q}^Q(\theta)\geq 0$ for all θ. ■

8.5 Proof of Proposition 4

(i) Combining the optimality conditions obtained under QF, and expressing everything in terms of effort yields that $e^Q(\theta)$ solves:

$$2\psi'(e) - e = \theta - h(\theta), \ \forall \ \theta \in \Theta. \tag{17}$$

Doing the same under RPM, $e^R(\theta)$ solves:

$$2\psi'(e) - e = \theta - 2h(\theta)\psi''(e), \ \forall \ \theta \in \Theta. \tag{18}$$

From **A2**, the right-hand side of (17) is greater than that of (18) and thus $e^Q(\theta) \geq e^R(\theta)$ with equality only at $\theta = \overline{\theta}$. ■

(ii) Combining the optimality conditions obtained under QF, and expressing everything in terms of output yields that $q^Q(\theta)$ solves:

$$q = \psi'(2q - \theta + h(\theta)), \forall \ \theta \in \Theta. \tag{19}$$

Doing the same under RPM, $q^R(\theta)$ solves:

$$q = \psi'(2q - \theta + h(\theta)) + h(\theta)\psi''(2q - \theta + h(\theta)), \forall \ \theta \in \Theta. \tag{20}$$

Note then that, for any h, we have:

$$\psi'(x + h) > \psi'(x) + h\psi''(x) \text{ when } \psi'''(\cdot) > 0$$

and

$$\psi'(x + h) < \psi'(x) + h\psi''(x) \text{ when } \psi'''(\cdot) < 0.$$

When $\psi'''(\cdot) > 0$, the right-hand side of (19) is greater than that of (20) and thus $q^R(\theta) \geq q^Q(\theta)$ with equality only at $\theta = \overline{\theta}$. This is the reverse when $\psi'''(\cdot) < 0$. ■

8.6 Proof of Proposition 5

(i) Given a generic $\theta \in \Theta$ and $\omega \in \mathcal{M}$, define the net consumers surplus by:

$$CS^\omega(\theta) \equiv \int_0^{q^\omega(\theta)} P\left(x, e^\omega(\theta), \theta\right) dx - P(q^\omega\left(\theta\right), e^\omega\left(\theta\right), \theta)q^\omega\left(\theta\right) = (q^\omega\left(\theta\right))^2/2$$

Straightforward algebraic manipulations yield $E_\theta[\Delta CS\left(\theta\right)] = E_\theta[(q^R(\theta) - q^Q(\theta))(q^R(\theta) + q^Q(\theta))]/2$. Since $q^R(\theta) \geq q^Q(\theta)$ (resp. ≤ 0) for all θ if $\psi'''(.) \geq 0$ (resp. ≤ 0) for all e, the claim then follows immediately. ■

(ii) The proof of this claim is based on a straightforward revealed preferences argument, so it will be omitted.

8.7 Proof of Proposition 6

First note that,

$$\Delta\Pi(\theta) = (\theta + e^R(\theta) - q^R(\theta))q^R(\theta) - (\theta + e^Q(\theta) - q^Q(\theta))q^Q(\theta) + \psi(e^Q(\theta)) - \psi(e^R(\theta))$$

can be rewritten using the definitions of $\left\{q^R(\theta), e^R(\theta)\right\}_{\theta\in\Theta}$ and $\left\{q^Q(\theta), e^Q(\theta)\right\}_{\theta\in\Theta}$ as:

$$\Delta\Pi(\theta) = (q^R(\theta))^2 - (q^Q(\theta))^2 - h(\theta)q^Q(\theta) \ + \psi(2q^Q(\theta) - \theta + h(\theta)) - \psi(2q^R(\theta) - \theta)$$

where $q^Q(\theta) = \psi'(2q^Q(\theta) - \theta + h(\theta))$.

First, using convexity of $\psi(.)$, we get:

$$\psi(2q^Q(\theta) - \theta + h(\theta)) - h(\theta)\psi'(2q^Q(\theta) - \theta + h(\theta)) \leq \psi(2q^Q(\theta) - \theta)$$

Therefore,

$$\Delta\Pi(\theta) \leq (q^R(\theta))^2 - (q^Q(\theta))^2 - \psi(2q^R(\theta) - \theta) + \psi(2q^Q(\theta) - \theta) \tag{21}$$

Now, notice that the function $\Psi(x) = x^2 - \psi(2x - \theta)$ is maximized at the first best output $q^*(\theta)$ and is concave by assumption.

The steps below conclude the proof.

Step 1. To begin with, consider the case $\psi'''(.) \leq 0$ so that $q^R(\theta) \leq q^Q(\theta) \leq q^*(\theta)$ for all θ. Then from equation (21) we get:

$$\Delta\Pi(\theta) \leq (q^R(\theta))^2 - (q^Q(\theta))^2 - \psi(2q^R(\theta) - \theta) + \psi(2q^Q(\theta) - \theta) \leq 0 \ \forall \ \theta \in \Theta$$

where the last equality follows from the concavity of $\Psi(.)$. Taking expectations yields $E_\theta[\Delta\Pi\,(\theta)] \leq 0$.

Step 2. Consider now $\psi'''(.) > 0$ so that $q^Q(\theta) < q^R(\theta) < q^*(\theta)$ for all $\theta < \bar{\theta}$. Then, define $g(y) = \psi(x+y) - y\psi'(x) - \psi(x)$. One can check that $g'''(y) = \psi'''(x+y)$. Therefore,

$$g(y) \geq g(0) + yg'(0) + \frac{y^2}{2}g''(0) = \frac{y^2}{2}g''(0)$$

Taking $x = 2q^Q(\theta) - \theta$ and $y = h(\theta)$, we get the following inequality:

$$\psi(2q^Q(\theta) - \theta + h(\theta)) - h(\theta)\psi'(2q^Q(\theta) - \theta) - \psi(2q^Q(\theta) - \theta) \geq \frac{h(\theta)^2}{2}\psi''(2q^Q(\theta) - \theta) \tag{22}$$

Inserting (22) into $\Delta\Pi(\theta)$ yields:

$$\Delta\Pi(\theta) \geq \left\{q^R(\theta)^2 - \psi(2q^R(\theta)-\theta) - (q^Q(\theta)^2 - \psi(2q^Q(\theta)-\theta))\right\} + \frac{h(\theta)^2}{2}\psi''(2q^Q(\theta)-\theta) \tag{23}$$

The second term in (23) is positive, the first one is also positive when $q^*(\theta) > q^R(\theta) > q^Q(\theta)$ for all $\theta < \bar{\theta}$ since $\Psi(.)$ is concave. Taking expectations yields $E_\theta[\Delta\Pi\,(\theta)] \geq 0$.

Finally, gathering steps 1 and 2 establishes the proof. ■

8.8 Proof of Corollary 7

By Proposition 6, a necessary condition for RPM to maximize constrained joint-profits is $\psi'''(e) > 0$ for all e. Since by Proposition 4 $\psi'''(e) > 0$ for all e implies $q^R(\theta) \geq q^Q(\theta)$ for all θ with equality at $\bar{\theta}$, the claim follows immediately by part (i) of Proposition 5. ■